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## DETERMINING THE COEFFICIENTS OF A HYDRODYNAMIC MODEL OF CAVITATING PUMPS OF LIQUID-PROPELLANT ROCKET ENGINES FROM THEIR THEORETICAL TRANSFER MATRICES

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The characterization of cavitating pumps of liquid-propellant rocket engines (LPRE) is an important problem because of the need to provide the pogo stability of liquid-propellant launch vehicles and the stability of liquid-propellant propulsion systems for cavitation oscillations. The development of a reliable mathematical model of LPRE cavitating pumps allows this problem to be resolved. The goal of this work is to determine the cavitation number and operating parameter dependences of the coefficients of a lumped-parameter hydrodynamic model of LPRE cavitating pumps from their theoretical transfer matrices obtained by a distributed-parameter model. The following coefficients are found as a function of operating parameters: the cavitation elasticity, the cavitation resistance, the cavity-caused disturbance transfer delay time, and the cavitation resistance distribution coefficient. The last two coefficients are new in the hydrodynamic model of cavitating pumps, and they were introduced when verifying the model using experimental and theoretical pump transfer matrices. Analyzing the cavitation resistance distribution coefficient as a function of operating parameters shows that it markedly decreases with increasing cavitation number. This testifies to that the location of the lumped cavity compliance is shifted from the mid position towards the pump inlet. Therefore, the assumption that the lumped cavity compliance is located in the middle of the attached cavity regardless of the cavitation number is not justified. The fact that the distribution coefficient as a function of cavitation number intersects the abscissa axis near a cavitation number of 0.25 may indicate the boundary of existence of attached cavities and thus the applicability boundary of the theoretical model. The disturbance transfer delay time as a function of cavitation number sharply increases at cavitation numbers of about 0.05. At cavitation numbers of about 0.25, it is close to a constant.

**Keywords:** *liquid-propellant rocket engine, inducer-equipped centrifugal pump, cavitation, hydrodynamic model, transfer matrix.*

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