

## FEATURES OF MATHEMATICAL SIMULATION OF GAS PATH DYNAMICS IN THE PROBLEM OF THE STABILITY OF LOW-FREQUENCY PROCESSES IN LIQUID-PROPELLANT ROCKET ENGINES

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One of the important problems in the designing of liquid-propellant rocket engines (LPREs) is the study of the stability of low-frequency processes in LPREs by mathematical simulation. In the low-frequency range, the dynamics of most of the LPRE components is described by ordinary differential equations (ODEs). The exception is the LPRE gas paths: a combustion chamber, a gas generator and gas lines, processes in which are described by delay equations. Since low-frequency oscillations in LPREs may be caused by the instability of processes in some of the LPRE systems, in the numerical study of stability the LPRE must be considered as a multi-loop dynamic system with potentially instable subsystems. An efficient method to study the stability of such systems in linear formulation is based on calculating the eigenfrequency spectrum of the operator matrix of a linear system of ODEs; however, that method is oriented to dynamic systems described by ODEs. To apply it to the analysis of the LPRE low-frequency stability, in the mathematical model of gas path dynamics one has to go from delay equations to ODEs. This paper addresses the problem of accounting for delays in the analysis of the LPRE low-frequency stability from the matrix spectrum. Schemes are constructed for approximate replacement of delay equations with ODEs based on approximating the delay element transfer function in a small parameter region by fractional rational functions and chains of functions. Different approximants of the delay element transfer function are considered and compared with one another. A rational approach to accounting for delays in the equations of LPRE gas path dynamics is proposed, and methodological recommendations on accounting for them are formulated. The results of this study may be used in simulating the low-frequency dynamics of gas paths and analyzing the stability of low-frequency processes in LPREs.

**Keywords:** *liquid-propellant rocket engine, dynamic system, Lyapunov stability, matrix spectrum, delay element, transfer function, approximants, frequency characteristics, oscillation eigenfrequencies and decrements.*

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