

SELF-VIBRATIONS OF A FUNCTIONALLY GRADED NANOCOMPOSITE CYLINDRICAL SHELL IN A SUPERSONIC GAS FLOW

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A model of geometrically nonlinear dynamic deformation of a functionally graded nanotube-reinforced composite cylindrical shell is derived. The shell is considered to be simply supported. Reddy's high-order shear deformation theory is used. Three projections of the displacements of the points of the middle surface and two rotation angles of the middle surface normal are the main unknowns of the model. The potential energy of geometrically nonlinear deformation of the cylindrical shell is obtained with account for shear. The three displacement projections and the two rotation angles of the middle surface normal are expanded by the normal modes of the cylindrical shell, including the axisymmetric modes. Using the assumed-mode method, a high-dimension nonlinear system of ordinary differential equation is derived to describe the nonlinear vibrations of the structure. The piston theory is used to describe the supersonic gas flow past the shell. The extended rule of mixture is used to obtain the mechanical properties of the nanocomposite. The characteristic exponents are calculated and a direct numerical integration of the linearized motion equations is used to analyze the dynamic stability of the trivial equilibrium. As shown by the numerical analysis, the trivial equilibrium loses stability due to the Hopf bifurcation. At the Hopf bifurcation point, a limit cycle, which describes traveling waves in the circumferential direction of the cylindrical shell, is originated. To analyze the limit cycle behavior in relation to the unperturbed pressure, use is made of the describing function method, in which the single harmonic approximation for self-vibrations is employed. The results obtained by the describing function method are compared with those of the direct numerical integration. The two methods give close results, thus demonstrating the adequacy of the describing function method in the study of self-vibrations.

Keywords: *functionally graded composite, cylindrical shell in a supersonic gas flow, high-order shear deformation theory, dynamic instability, self-vibrations*

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