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UNIVERSAL SPLINE-PERTURBED DISTRIBUTION

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This paper considers the problem of probability distribution construction for a random variable from known numerical characteristics. The problem is of importance in determining the parametric reliability of engineering systems when the numerical characteristics (in particular, the bias and the kurtosis) of an output parameter (state variable) are determined by analytical methods and its distribution must be recovered. This may be done using a four-parameter universal distribution, which allows one to cover certain ranges (preferably, as wide as possible) of the bias and kurtosis coefficients using a single analytical form. The most familiar universal distribution is Gram-Charlier's, which is a deformation of the normal distribution obtained using a Chebyshev-Hermite orthogonal polynomial expansion. However, in the general case, Gram-Charlier's distribution function is not a steadily increasing one. For some combinations of the bias and kurtosis coefficients, the density curve may exhibit negative values and multiple modes. Because of this, a search for other universal distributions to cover wider ranges of the bias and kurtosis coefficients is of current importance.

The paper analyzes a method of universal probability distribution construction by multiplying the normal density by a perturbing polynomial in the form of a spline (referred to as the spline-perturbed distribution). The idea of a distribution of this type was proposed earlier to account for a nonzero bias coefficient. The spline is constructed based on Hermite's interpolating polynomials of the third degree with two knots, which have a minimum of parameters and possess a locality property The basic distribution is constructed for a four-knot spline.

The paper further develops and generalizes the spline-perturbed distribution to nonzero bias and kurtosis coefficients. Two cases are considered. The first case is a composition of two splines that have four and five knots, respectively. The former and the latter allow one to account for the bias and the kurtosis, respectively. Integral equations are obtained to find the values at the knots of both splines and construct the distribution. The second case is more general and uses one five-knot Hermite spline. The paper shows a way to construct a generalized spline-perturbed distribution without any negative density values or any multiple modes. The knot points are chosen using an enumerative technique. Conditions for the absence of negative density values and multiple nodes are identified.

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