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CONSTRUCTION AND ANALYSIS OF UNIVERSAL 2D DISTRIBUTIONS WITH A BOUNDED RECTANGULAR VARIATION DOMAIN

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When solving parametric reliability problems, one often has to construct distributions of statistical data to find the probability of containment in the operability region. This paper considers the problem of 2D statistical ensemble fitting. The use of a 2D normal distribution in statistical data description is not always justified because statistical ensembles rather frequently (at the level of marginal components and a stochastic relationship between them) have properties different from the normal case. From a practical standpoint, it is desirable for researchers to describe 2D statistical ensembles with the use of universal distributions, which allow one to cover a wide range of source data using a single analytical form. In the process of fitting, account should be made of bounded ranges of random variables.

The paper considers who universal distribution construction methods, which are based on 1D orthogonal Jacobi polynomial expansions. In these distributions, the random variable range is a rectangle. In the first method, a 2D distribution is constructed using a direct expansion in the 1D Jacobi polynomials. A 2D Jacobi distribution function and regression lines are obtained, and methods to fit it are considered. In theory, a distribution obtained in this way can be used, up to the fourth order inclusive, for marginal and even reduced moments different from the normal case. However, its real capabilities are limited to values of reduced moments (1D and even) that differ from the normal case only very slightly. Otherwise, the probability surface may enter negative ranges with the occurrence of multiple modes.

The second way to construct a 2D distribution is to use a normal copula and 1D Jacobi distributions as components. The resulting 2D distribution allows one to deal with 1D distributions different from the normal case and linear correlation. This approach is justified because, according to research data, it is a linear stochastic relationship that relates a significant part of 2D statistical ensembles, and marginal distributions deviate from the normal case. Regression lines of a distribution of this kind are obtained, and it is shown that they are curved because marginal distributions differ from the normal one. The paper considers the practical example of fitting a 2D ensemble of characteristics of a liquid-propellant rocket engine some components of which are related via a linear stochastic relationship (the parameters that characterize a nonlinear stochastic relationship proved to be insignificant) and have 1D distributions different from the normal one. The fitted and observed frequencies are in rather good agreement. It is shown that a distribution based on a normal copula is more universal, and it is recommended for practical calculations.

Keywords: 2D distribution, orthogonal Jacobi polynomials, Jacobi distribution, normal copula, fitting.

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