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USE OF TWO-DIMENSIONAL NORMAL COPULA IN PARAMETRIC RELIABILITY ESTIMATION MODELS

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In most cases, determining the parametric reliability of the mechanical systems (MSs) of a launch vehicle (LV) at the design stage can be reduced to one- and two-dimensional models. The use of the normal distribution in such models is not always justified because the MS parameters often obey distribution laws distinct from the normal one. This paper demonstrates that the LV MS parametric reliability can be estimated using a two-dimensional normal copula constructed on the basis of one-dimensional generalized lambda distributions, which show a considerable flexibility. The construction and features of a normal copula of this type are considered; in particular, expressions for the distribution density, regression lines, and the distribution function are presented. Such a distribution allows one to account for the difference of marginal distributions from the normal one and a linear correlation between the components (a linear correlation between the MS parameters is observed in 70 percent of cases). It is shown how the normal copula parameter that characterizes a linear correlation between random variables can be obtained using the method of moments.

In this paper, expressions for determining the LV MS parametric reliability are derived using the normal copula constructed on the basis of one-dimensional generalized lambda distributions. With their help, it is shown that accounting for both the difference of marginal distributions of random variables from the normal one (first of all, the skew and the kurtosis) and for a linear correlation between them offers a more accurate prediction of the MS reliability in comparison with the normal case. Accounting for a nonlinear correlation between the MS parameters (a modified Farlie–Gumbel–Morgenstern copula is used for comparison) does not either result in any significant deviation of the reliability index from the values obtained with the use of the normal copula considered.

The practical use of the normal copula considered is demonstrated by the example of estimating the probability of the propellant of an LV stage being sufficient for a trouble-free cutoff of the propulsion system.

Keywords: *launch vehicle, probability of failure-free operation, state variables, normal copula, generalized lambda distribution.*

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