

**Nonlinear oscillations of a sandwich plate with a 3D-printed honeycomb core**

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A three-layer sandwich plate with a FDM-printed honeycomb core made of polycarbonate is considered. The upper and lower faces of the sandwich are made of a carbon fiber-reinforced composite. To study the response of the sandwich plate, the honeycomb core is replaced with a homogeneous layer with appropriate mechanical properties. To verify the honeycomb core model, a finite-element simulation of the representative volume of the core was performed using the ANSYS software package. A modification of the high-order shear theory is used to describe the structure dynamics. The assumed-mode method is used to simulate nonlinear forced oscillations of the plate. The Rayleigh–Ritz method is used to calculate the eigenfrequencies and eigenmodes of the plate, in which the displacement of the plate points during nonlinear oscillations are expanded. This technique allows one to obtain a finite-degree-of-freedom nonlinear dynamic system, which describes the oscillations of the plate. The frequency response of the system is calculated using the continuation approach applied to a two-point boundary value problem for nonlinear ordinary differential equations and the Floquet multiplier method, which allows one to determine the stability and bifurcations of periodic solutions. The resonance behavior of the system is analyzed using its frequency response.

The proposed technique is used to analyze the forced oscillations of a square three-layer plate clamped along the contour. The results of the analysis of the free oscillations of the plate are compared with those of ANSYS finite-element simulation, and the convergence of the results with increasing number of basis functions is analyzed. The comparison shows that the results are in close agreement. The analysis of the forced oscillations shows that the plate executes essentially nonlinear oscillations with two saddle-node bifurcations in the frequency response curve, in which the periodic motion stability of the system changes. The nonlinear oscillations of the plate near the first fundamental resonance are mostly monoharmonic. They may be calculated using the describing function method.

**Keywords:** sandwich plate, honeycomb core, homogenization, nonlinear dynamic system, frequency response.

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