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REDUCING THE DIMENSION OF A NONLINEAR DYNAMIC SYSTEM TO SIMULATE A MULTI-WALLED NANOTUBE

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A system of nonlinear partial differential equations is derived to describe the vibrations of a multi-walled nanotube. The system reduces to a nonlinear dynamic system with a large number of degrees of freedom (DOFs). To reduce its dimension, the nonlinear modal analysis method is used to give 2-DOF dynamic system, which is studied by the asymptotic multiple scale method. This gives a system of modulation equations, whose fixed points describe the free vibrations of the nanotube. The fixed points are described by nonlinear algebraic equations, whose solutions are given on a backbone curve. Use is made of the Sanders–Koiter shell model to describe the nonlinear deformation of the nanotube and Hook's nonlocal anisotropic law to simulate its vibrations. Notice that the elastic constants of the nanotube walls differ. The nanotube model is a system of nonlinear ordinary differential equations, which is obtained by applying the weighed residuals method to the nonlinear partial equations. Three types of nonlinearities are accounted for in the nanotube model. First, the Van der Waals forces are nonlinear functions of the radial displacements. Second, the displacements of the nanotube walls are assumed to be moderate, which is described by a geometrically nonlinear model. Third, since the resultant forces are nonlinear functions of the displacements, the use of natural boundary conditions in the weighted residuals method results in additional nonlinear terms. A finite-DOF nonlinear dynamical system is derived. The free nonlinear vibrations of the nanotube are analyzed. The calculated results are shown on a backbone curve.

Keywords*: reduced order modeling, nanotube, Hook's nonlocal anisotropic law, finite degree of freedom nonlinear dynamical system, multi-mode invariant manifold.*

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