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PHASE AMBIGUITY RESOLUTION IN RELATIVE DISPLACEMENT MEASUREMENT BY MICROWAVE INTERFEROMETRY

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Рассматривается сверхвысокочастотный интерференционный метод измерения, который широко используется для определения перемещения в различных технических приложениях. Целью данной статьи является разработка методики устранения фазовой неопределенности при измерении относительного перемещения механических объектов с помощью двухзондовой реализации интерференционного метода. Для определения неразвернутой фазы из квадратурних сигналов предложено использовать меньший корень биквадратного уравнения, связывающего неизвестный коэффициент отражения с токами полупроводниковых детекторов, соединенных с зондами. Найдены диапазоны коэффициента отражения и действительной неразвернутой фазы, в которых определяемая таким образом неразвернутая фаза является кажущейся. Показано, что погрешность определения перемещения, обусловленная отличием кажущейся неразвернутой фазы от действительной, отлична от нуля только для достаточно больших коэффициентов отражения и не превышает нескольких процентов от длины волны зондирующего электромагнитного излучения. Установлено, что для размеров контролируемого объекта и расстояний между объектом и антенной, для которых выполняется приближение плоской волны, предложенная методика позволяет определять размах колебаний с точностью порядка нескольких десятых процента даже при размахе, в несколько раз превышающем длину волны зондирующего электромагнитного излучения. Предложенная методика может быть использована при разработке датчиков перемещения с упрощенной аппаратной реализацией.

Розглядається надвисокочастотний інтерференційний метод вимірювання, який широко використовується для визначення переміщення в різноманітних технічних застосуваннях. Метою цієї статті є розробка методики усунення фазової невизначеності при вимірюванні відносного переміщення механічних об’єктів за допомогою двозондової реалізації інтерференційного методу. Для визначення нерозгорнутої фази з квадратурних сигналів запропоновано використовувати менший корінь біквадратного рівняння, що пов’язує невідомий коефіцієнт відбиття зі струмами з’єднаних із зондами напівпровідникових детекторів. Знайдено діапазони коефіцієнта відбиття й дійсної нерозгорнутої фази, в яких визначена таким чином нерозгорнута фаза є позірною. Показано, що похибка визначення переміщення, зумовлена різницею між позірною й дійсною нерозгорнутою фазою, відрізняється від нуля тільки для досить великих коефіцієнтів відбиття та не перевищує декількох процентів від довжини хвилі зондуючого електромагнітного випромінювання. Встановлено, що для розмірів контрольованого об’єкта й відстаней між об’єктом та антеною, для яких виконується наближення плоскої хвилі, запропонована методика дозволяє визначати розмах коливань з точністю порядку декількох десятих процента навіть при розмасі, що у декілька разів перевищує довжину хвилі зондуючого електромагнітного випромінювання. Запропонована методика може бути використана при розробці давачів переміщення зі спрощеною апаратною реалізацією.

This paper addresses microwave interferometry, which is widely used for displacement determination in various engineering applications. The aim of this paper is to develop a technique for phase ambiguity resolution in measurements of the relative displacement of mechanical objects using a two-probe implementation of microwave interferometry. To determine the wrapped phase from the quadrature signals, it is suggested to use the smaller positive root of the biquadratic equation that relates the unknown reflection coefficient to the currents of the semiconductor detectors connected to the probes. The reflection coefficient range and actual wrapped phase range in which the wrapped phase determined in this way is an apparent one are found. It is shown that the displacement determination error that is due to the difference of the apparent and the actual wrapped phase is nonzero only for sufficiently large reflection coefficients and does not exceed a few percent of the operating wavelength. It is found that for the target dimensions and the target–antenna distances for which the plane wave approximation holds, the proposed technique determines the vibration peak-to-peak amplitude to within several tenths of a percent even for peak-to-peak amplitudes several times greater than the operating wavelength. The proposed technique may be used in the development of displacement sensors with a simplified hardware implementation.

Keywords: *phase ambiguity, complex reflection coefficient, electrical probe, semiconductor detector, waveguide section, interprobe distance.*

Microwave measurements are widely used in the determination of various parameters such as distance, displacemet, speed, dielectric permittivity, etc. Microwave interferometry is an ideal means in terms of the development of motion sensors [1]. This is due to its ability to provide fast noncontact measurements and its applicability to dusty or smoky environments (as distinct from laser Doppler sensors [2 – 4] or vision-based systems using digital image processing techniques [5]). An important advantage over radar methods (both traditional pulse ones and recently developed continuous-wave step-frequency ones [6, 7]) is its simple hardware implementation. In microwave interferometry, the displacement of the object under measurement (target) is extracted from the phase shift between the signal reflected from the target and the reference signal, i. e. from the phase of the complex reflection coefficient. A characteristic feature of such measurements is phase ambiguity. This is due to the fact that the complex reflection coefficient phase, which contains information on the parameters to be measured, in the general case can only be determined to within 2π. In displacement measurements, this phase ambiguity can be resolved by using two quadrature signals in combination with a phase unwrapping method. At present, the usual way to form the quadrature signals is to use special hardware incorporating a power divider and a phase-detecting processor, which is an analog [8] or a digital [9] quadrature mixer. The advantage of the latter is that it can eliminate or at least minimize the nonlinearity of the phase response of the former, which is caused by its phase and amplitude unbalances and by the dc voltage offset in the amplifier. However, this is achieved at the expense of the far more complex design of the meter, including the use of an intermediate frequency.

An intermediate frequency is also used in the method proposed in [10]. In that method, the probing microwave signal is modulated with an intermediate-frequency signal, whose wavelength is much longer than that of the probing signal. The modulated signal reflected from the target is mixed with the nonmodulated reference signal. As a result of the mixing, intermediate-frequency quadrature signals are formed followed by their extraction. The phase of the reflected signal is found by mathematical treatment of the quadrature signals, and the target displacement is determined from this phase. Due to the use of an intermediate frequency, the hardware implementation of the above-described method is rather complex and requires such devices as a phase-shift modulator, which phase-modulates the probing signal with the intermediate frequency one; a digital-to-analog converter, in which the modulating intermediate-frequency signal is formed; a balance mixer, in which the modulated reflected signal interferes with the nonmodulated reference signal and at the output of which the intermediate-frequency quadrature signals are extracted; and two directional couplers with matched loads to direct the microwave oscillator signal to the phase-shift modulator and the reflected signal to the balance mixer.

As can be seen from the aforesaid, the above-considered traditional methods of phase ambiguity resolution in displacement measurement by microwave interferometry are rather complex in hardware implementation. On the other hand, information on the phase of the complex reflection coefficient is also contained in the electric field amplitude of the standing wave between the emitter and the target, which can be measured using an electrical probe and a semiconductor detector connected thereto. The hardware implementation of probe measurements is much simpler. Since the publication of the classic text by Tischer [11], there has been general agreement that at least three probes are needed for phase ambiguity resolution by determining or eliminating the unknown reflection coefficient without recourse to detector current differentiation [12 – 18]. Of interest is to consider whether the number of probes can be reduced to two because this would allow one:

– to simplify the design of the waveguide section;

– to simplify the manufacture of the meter because only one interprobe distance must be held to close tolerances;

– to reduce the parasitic effect of multiple reflections between the probes;

– to reduce the number of channels of the analog-to-digital converter thus increasing the upper bound of the sampling frequency.

The aim of this paper is develop a technique for phase ambiguity resolution in mechanical object displacement measurement using two electrical probes.

Consider two probes 1 and 2 connected to square-law semiconductor detectors. The probes are placed $\lambda_g/8$ apart (here, λ_g is the guided operating wavelength) in a waveguide section between a microwave oscillator and a target, probe 2 being closer to the target. A measurement schematic is shown in Fig. 1.

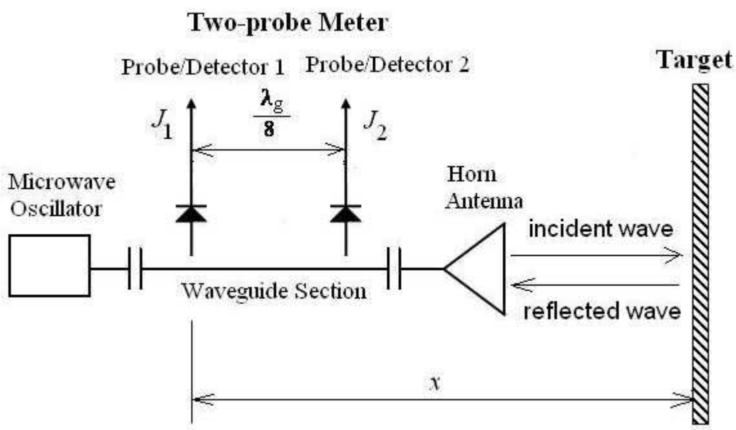

Fig. 1

The detector currents J_1, J_2 normalized to their values in the absence of a reflected wave are expressed in terms of the magnitude R and phase ψ of the complex reflection coefficient at the location of probe 1

$$J_1 = 1 + R^2 + 2R\cos\psi \ , \tag{1}$$

$$J_2 = 1 + R^2 + 2R\sin\psi \ . \tag{2}$$

Information on the distance x between the target and probe 1 in contained in the phase of the complex reflection coefficient

$$\psi = \frac{4\pi x}{\lambda_0} + \phi , \tag{3}$$

where λ_0 is the free-space operating wavelength and ϕ is the phase component that is governed by the waveguide section and horn antenna geometry and the phase shift caused by the reflection and does not depend on the distance x.

Let it be desired to find the displacement Δx of the target at time t relative to its initial position $x(t_0)$. As indicated above, for phase ambiguity resolution in relative displacement determination it is sufficient to have the quadrature signals $\cos\psi$ и $\sin\psi$. According to Eqs. (1) and (2), these signals are expressed in terms of the unknown magnitude of the reflection coefficient as follows

$$\cos\psi = \frac{a_1 - R^2}{2R}, \tag{4}$$

$$\sin\psi = \frac{a_2 - R^2}{2R}, \tag{5}$$

where $a_1 = J_1 - 1$ and $a_2 = J_2 - 1$.

The following biquadratic equation in R results from Eqs. (4) and (5)

$$R^4 - (a_1 + a_2 + 2)R^2 + \frac{a_1^2 + a_2^2}{2} = 0. \tag{6}$$

This equation has two positive roots (the plus sign before the radical corresponds to the root R_1, and the minus sign corresponds to the root R_2)

$$R_{1,2} = \left[\frac{a_1 + a_2 + 2}{2} \pm \sqrt{\frac{(a_1 + a_2 + 2)^2}{4} - \frac{a_1^2 + a_2^2}{2}}\right]^{1/2}.$$

Clearly one of these roots is extraneous. So, the phase ambiguity resolution problem reduces to the choice between the root R_1 and the root R_2.

An explicit expression for the extraneous root may be obtained by rearranging the absolute term of Eq. (6). From Eqs. (4) and (5) we have

$$a_1^2 = R^4 + 2R^3\cos\psi + 4R^2\cos^2\psi, \tag{7}$$

$$a_1^2 = R^4 + 2R^3\sin\psi + 4R^2\sin^2\psi. \tag{8}$$

Substituting Eqs. (7) and (8) into the expression for the absolute term of Eq. (6) gives

$$\frac{a_1^2 + a_2^2}{2} = R^2\left[R^2 + 2\sqrt{2}R\sin(\psi + \pi/4) + 2\right]. \tag{9}$$

Since the absolute term of a quartic equation is equal to the product of its roots, it follows from Eq. (9) that the positive extraneous root R_{ext} of Eq. (6) is

$$R_{ext} = \left[R^2 + 2\sqrt{2}R\sin(\psi + \pi/4) + 2\right]^{1/2}. \tag{10}$$

Let us find the condition under which the inequality $R_{ext} \geq R$ is satisfied. It follows from Eq. (10) that this condition is

$$\sin\left(\psi + \frac{\pi}{4}\right) \geq -\frac{1}{\sqrt{2}R}. \tag{11}$$

This inequality is satisfied at any value of the phase ψ if $R \le 1/\sqrt{2}$. Since $R_1 \ge R_2$, in this case the reflection coefficient magnitude R will always be given by the root R_2. In the case $R > 1/\sqrt{2}$, the condition of (11) will not be necessarily satisfied. Because of this, the reflection coefficient magnitude R will be given by the root R_2 if the condition of (11) is satisfied; otherwise it will be given by the root R_1.

First consider the case $R \le 1/\sqrt{2}$. In this case, the reflection coefficient magnitude R is unambiguously determined from Eq. (6) as its root R_2, and thus $\cos\psi$ and $\sin\psi$ are unambiguously determined from Eqs. (4) and (5). If $\cos\psi$ and $\sin\psi$ are known, the displacement Δx of the target at time t_n, $n = 0, 1, 2, ...,$ from its initial position $x(t_0)$ can be found by the following phase unwrapping algorithm [19]

$$\varphi(t_n) = \begin{cases} \arctan \dfrac{\sin\psi(t_n)}{\cos\psi(t_n)}, & \sin\psi(t_n) \ge 0, \cos\psi(t_n) \ge 0, \\ \arctan \dfrac{\sin\psi(t_n)}{\cos\psi(t_n)} + \pi, & \cos\psi(t_n) < 0, \\ \arctan \dfrac{\sin\psi(t_n)}{\cos\psi(t_n)} + 2\pi, & \sin\psi(t_n) < 0, \cos\psi(t_n) \ge 0, \end{cases} \tag{12}$$

$$\Delta\varphi(t_n) = \varphi(t_n) - \varphi(t_{n-1}), \tag{13}$$

$$\theta(t_n) = \begin{cases} 0, \ n = 0, \\ \theta(t_{n-1}) + \Delta\varphi(t_n), & |\Delta\varphi(t_n)| \le \pi, \quad n = 1, 2, ..., \\ \theta(t_{n-1}) + \Delta\varphi(t_n) - 2\pi \,\mathrm{sgn}[\Delta\varphi(t_n)], & |\Delta\varphi(t_n)| > \pi, \ n = 1, 2, .., \end{cases} \tag{14}$$

$$\Delta x(t_n) = \frac{\lambda_0}{4\pi}\theta(t_n), \quad n = 0, 1, 2, ..., \tag{15}$$

where φ and θ are the wrapped and the unwrapped phase, respectively.

Now consider the case $R > 1/\sqrt{2}$. In this case, R_2 will be equal to R only for the values of ψ that satisfy the condition of (11). However, as will be shown below, the displacement can also be determined to sufficient accuracy using the root R_2 as the reflection coefficient magnitude. It follows from the condition of (11) that the root R_2 will be extraneous if $\sin(\psi + \pi/4) < -1/\sqrt{2}\,R$. In terms of the wrapped phase φ, this condition becomes

$$\frac{3\pi}{4} + \arcsin\frac{1}{\sqrt{2}R} < \varphi < \frac{7\pi}{4} - \arcsin\frac{1}{\sqrt{2}R},$$

whence it follows that the wrapped phase that corresponds to the condition $\sin(\psi + \pi/4) < -1/\sqrt{2}\,R$ lies in the third quadrant.

Let us find the phase error that is introduced when the extraneous root R_{ext} is used as the reflection coefficient magnitude. In this case, Eqs. (4) and (5) will give the apparent values $\cos\psi_{ap} = \left(a_1 - R_{ext}^2\right)/2R_{ext}$ and $\sin\psi_{ap} = \left(a_2 - R_{ext}^2\right)/2R_{ext}$, which on substitution into Eq. (12) will give the apparent wrapped phase φ_{ap}. The final expression for the apparent wrapped phase is ‘

$$\varphi_{ap} = \arctan\frac{1 + R\cos\varphi}{1 + R\sin\varphi} + \pi \ .$$

The use of the apparent wrapped phase φ_{ap} instead of the actual wrapped phase φ introduces the phase error $\Delta\varphi_{er}(\varphi, R) = \varphi_{ap} - \varphi$. The function $\Delta\varphi_{er}(\varphi, R)$ possesses the following properties:

– is antisymmetric in φ about $\varphi = \frac{5\pi}{4}$;

– becomes zero at $\varphi = \frac{3\pi}{4} + \arcsin\frac{1}{\sqrt{2}R}$, $\varphi = \frac{5\pi}{4}$, and $\varphi = \frac{7\pi}{4} - \arcsin\frac{1}{\sqrt{2}R}$;

– at a fixed φ, increases in magnitude with R;

– at a fixed R, has a negative minimum at $\varphi_1 = \frac{3\pi}{4} + \arcsin\frac{\sqrt{2}(1 + R^2)}{3R}$ and a positive maximum at $\varphi_2 = \frac{7\pi}{4} - \arcsin\frac{\sqrt{2}(1 + R^2)}{3R}$, which are equal in magnitude by virtue of the antisymmetry of the function.

It follows from these properties that the greatest possible phase error $\Delta\varphi_{er\max}$ is reached at $R = 1$ and is equal to

$$\Delta\varphi_{er\max} = \arctan\frac{\sqrt{2} + 1}{\sqrt{2} - 1} + \arcsin\frac{2\sqrt{2}}{3} - \frac{3\pi}{4}.$$

As can be seen from the algorithm of (12) – (15), the displacement determination error is governed by the phase error only at the initial and the current measurement point because the errors at the intermediate points cancel one another. Because of this, the greatest possible displacement determination error $\Delta x_{er\max}$ will be reached at $R = 1$ in the case where the initial measurement point corresponds to one extremum of the function $\Delta\varphi_{er}(\varphi)$ and the current measurement point corresponds to the other. As follows from the aforesaid, this error will be

$$\Delta x_{er\max} = \frac{\lambda_0}{4\pi} 2\Delta\varphi_{er\max} = 0{,}044\lambda_0 \,. \tag{16}$$

As can be seen from Eq. (16), the greatest possible error $\Delta x_{er\max}$ is about 4.4 % of the free-space operating wavelength λ_0 (notice that this is the worst-case error, which occurs when the reflection coefficient magnitude is equal to unity, the initial measurement point corresponds to one extremum of the function $\Delta\varphi_{er}(\varphi)$, and the current measurement point corresponds to the other). So the proposed phase ambiguity resolution technique, in which the reflection coefficient magnitude is taken to be equal to the smaller positive root of Eq. (6), allows the displacement to be determined to sufficient accuracy at any value of the reflection coefficient magnitude.

The above-described phase ambiguity resolution technique is based on the assumption that both the incident wave and the reflected wave are plane. However, for target dimensions comparable with the operating wavelength the reflected wave may be considered as plane only within some distance, which in the general case introduces a measurement error.

To relate the displacement measurement error to the antenna–target distance, measurements were conducted using a set-up, which comprised a measuring waveguide section with two probes installed therein and two semiconductor detectors connected to the probes, a horn antenna mounted at the end of the waveguide section, a microwave oscillator, and an analog-to-digital converter. The target was put into a reciprocal motion using a crank mechanism. The double amplitude of the target was 150 mm. The measurements were made at 9.76 GHz, which corresponds to a free-space operating wavelength of 3.07 cm. The semiconductor detector currents were measured and recorded using the analog-to-digital converter, and the relative displacement of the target was determined from the detector currents.

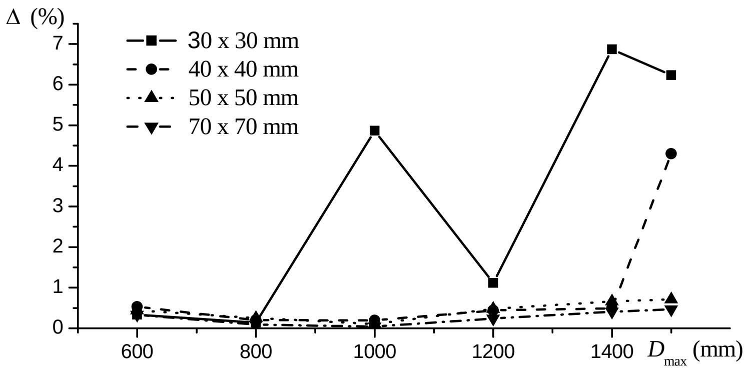

Fig. 2

Fog. 2 shows the relative error Δ of double amplitude determination versus the maximum distance $D_{\max}$ between the target and the horn antenna for targets in the form of 30×30 mm, 40×40 mm, 50×50 mm, and 70×70 mm metal squares. As can be seen from the figure, there exists a threshold distance beyond which the double amplitude error sharply increases, and this threshold distance increases with the target size. Within the threshold distance, the error depends only slightly on the distance and does not exceed 1 %.

This behavior of the error is due to the features of the phase unwrapping algorithm employed. This algorithm is based on the assumption that between two successive measurements the phase of the reflection coefficient, which is given by Eq. (3), changes in magnitude less than by π.

Let the measurement time step be Δt. In the assumption that the target executes a harmonic motion of amplitude A and frequency f_{vib}, the maximum displacement of the target in the time Δt will be $2\pi f_{vib} A \Delta t$, and in the plane wave approximation the maximum phase change in that time will be $8\pi^2 f_{vib} A \Delta t / \lambda_0$. Hence the algorithm applicability condition is

$$\frac{8\pi^2 f_{vib} A\Delta t}{\lambda_0} + \Delta\varphi < \pi \qquad (17)$$

where $\Delta\varphi$ is the difference of the actual phase change and the phase change in the plane wave approximation.

As the distance to the target increases, the reflected wave that returns to the antenna differs more and more from a plane wave, thus increasing the error of phase determination in the plane wave approximation. Eventually, there comes a time where this error $\Delta\varphi$ increases to an extent that the condition of (17) is no longer satisfied. As this takes place, the displacement determination error increases in a stepwise manner. This can be seen in Fig. 3, which shows the time dependence of the measured relative displacement of a 40×40 mm square target at D_{max} = 600 mm (the double amplitude error is less than 1 %) and 1,500 mm (the error increases sharply up to 4.3 %). As illustrated, at D_{max} = 1,500 mm the measured displacement shows jumps.

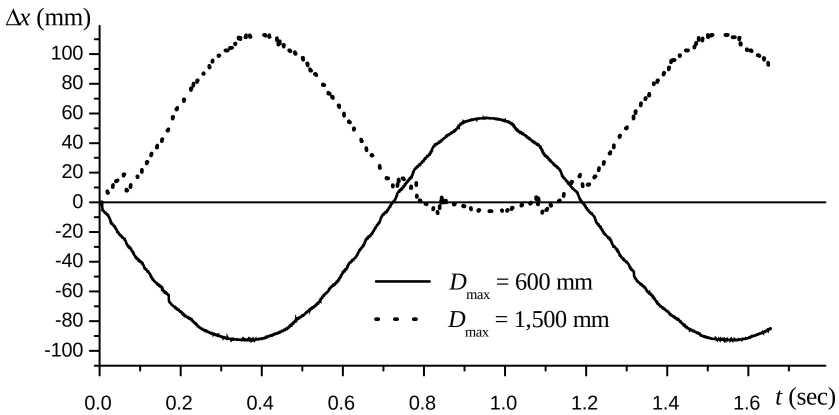

Fig. 3

So there exist a threshold distance between the antenna and the target beyond which the target double amplitude determination error increases in a stepwise manner, while within this threshold distance the error does not exceed several tenths of one percent even for a double amplitude several times as large as the operating wavelength.

The proposed technique may be used in the development of displacement meters with simplified hardware implementation.

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