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SYNTHESIS OF PARAMETRICALLY ROBUST CONTROLLER WITH EXTENDED STATE OBSERVER

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Розглянуто завдання керування параметрично-невизначеною динамічною системою з використанням спостерігача розширеного вектора стану. Система керування побудована за двоконтурною схемою, де зовнішній контур забезпечує виконання обраного критерію керування вектором стану, а внутрішній контур реалізує компенсацію або зменшення впливу сумарного вектора еквівалентного збурення.

Метою дослідження є розробка процедури синтезу спостерігача розширеного вектора стану з урахуванням параметричної невизначеності об'єкта керування та вимог до замкнутого контуру комбінованої системи керування, заданих у частотній області.

Під час проведення досліджень використано методи теорії керування, робастного керування та комп'ютерного моделювання.

Вплив параметричної невизначеності об'єкта керування на його керований рух представлений як структуроване збурення, що описується у формі блочно-діагональної матриці. Поняття структурованих сингулярних чисел використано при визначенні міри робастности системи.

З використанням методології оптимізації структурованих сингулярних чисел запропонована процедура синтезу спостерігача розширеного вектора стану при розгляді замкнутого контуру комбінованої системи керування з урахуванням параметричної невизначеності її математичної моделі. Вимоги, що забезпечують задану якість та стійкість замкнутого контуру системи керування з урахуванням спектральних властивостей збурень та перешкод вимірювачів, сформульовано в частотній області з використанням частотно-залежних вагових функцій. Для синтезу комбінованих регуляторів з урахуванням мінімізації структурованих сингулярних чисел розроблено алгоритм D-G-L-K ітерацій. Ефективність запропонованого підходу проілюстрована чисельним прикладом. Наведено рекомендації щодо синтезу параметрично робастних комбінованих регуляторів.

Практична цінність отриманих результатів полягає в тому, що розроблена процедура дозволяє зменшити консерватизм робастних комбінованих систем керування та, як наслідок, підвищити якість керування у випадку параметричної невизначеності об'єкта керування.

***Ключові слова**: спостерігач розширеного вектора стану, параметрична невизначеність, робастність, структуроване сингулярне число.*

This paper considers the problem of control of a parametrically uncertain dynamic system using an extended state observer. The control system is implemented using a two-loop scheme, where the outer loop ensures the fulfillment of the selected criterion for controlling the state vector, and the inner loop compensates for or reduces the influence of the vector of the total equivalent disturbance.

The goal of the study is to develop a procedure for synthesizing an extended state observer taking into account the parametric uncertainty of the plant and the requirements for the closed loop of the combined control system specified in the frequency domain.

Methods of control theory, robust control, and computer modeling are used for this study.

The effect of the parametric uncertainty of the plant on its controlled motion is presented as a structured disturbance described in the form of a block-diagonal matrix. The concept of structured singular values is used to determine the measure of the system's robustness.

Using the methodology of optimization of structured singular values, a procedure is proposed for synthesizing the extended state observer when considering a closed loop of a combined control system taking into account the parametric uncertainty of its mathematical model. The requirements ensuring the specified performance and stability of the closed loop of the control system taking into account the spectral properties of disturbances and sensor noise are formulated in the frequency domain using frequency-dependent weighting functions. For the synthesis of combined controllers based on the minimization of structured singular values, a D-G-L-K iteration algorithm is developed. The efficiency of the proposed approach is illustrated by a numerical example. Recommendations are given for the synthesis of parametrically robust combined controllers. The practical value of the obtained results lies in the fact that the procedure developed reduces the conservatism of robust combined control systems and, as a result, improves the control performance in the case of parametric uncertainty of the plant.

***Keywords:** extended state observer, parametric uncertainty, robustness, structured singular value.*

Introduction. Mathematical models are usually used to synthesize algorithms for controlling dynamic objects. In practice, it is difficult to avoid inaccuracies of such models. In this regard, the control system must be robust to these uncertainties [1, 2].

One of the efficient approaches to solving control problems with uncertainties is based on the reconstruction of the vector of the total equivalent disturbances, which, in addition to external disturbances, also includes internal disturbances caused by inaccuracies in the mathematical description of the plant. Such control can be implemented using a dual-loop scheme with the possibility of dividing the loops according to control objectives: the external loop is responsible for achieving the tracking or stabilization goals, and the internal one has to compensate for or reduce the influence of the disturbances [3–5]. The success of this approach is largely dependent on the quality of the disturbance reconstruction.

An extended state observer (ESO) can estimate different disturbances [6]. The paper [7] shows the significant capabilities of such an observer for estimating various typical disturbances. Still, the quality of the estimates in the presence of sensor noise largely depends on the observer's tuning. In article [8], it was proposed that the observer gains be selected based on the standard forms of pole placement. In this case, the only adjustable parameter is the observer's bandwidth, a certain choice that allows for a compromise between the accuracy of disturbance estimation and the sensitivity to sensor noise. Even though the efficiency of this approach is illustrated by the successful solution of some practical tasks [9–12], the optimality of this choice of the observer gains is not addressed. In this regard, the problem of finding the parameters of such an observer, optimal in one sense or another, taking into account the features of the problem of estimation and compensation of disturbances, is of interest.

The article [13] uses a fairly flexible representation of such a disturbance, which is suitable for taking into account both structural and parametric uncertainties in the controller design stage. However, in this case, the disturbance is completely unstructured, which leads to a significant conservatism of estimates in the case of structured disturbances, and in particular parametric uncertainties. Therefore, it is of interest to improve the procedures for the synthesis of such combined controllers in terms of accounting information about the parametric uncertainty of the plant.

Problem statement. Let us consider the problem of synthesizing a combined control system with an ESO, taking into account the parametric uncertainty of the plant.

The model of the plant is given by a nonlinear differential equation of n-th order in the following form

$$y^{(n)} = f(t, y, \dot{y}, \ldots, y^{(n-1)}, w_v) + b \ \ , \tag{1}$$

where y is the measured output; u is the scalar input (control); f is the function describing the internal dynamics of the plant and the external disturbances w_v; b is a parameter.

Equation (1) has the following representation in the of state space form

$$\dot{X}_0 = A_0 X_0 + B_0 \ u + B_0 \ f, Y_0 = C_0 X_0, Y_I = C_I X_0 + D_0 \xi, \tag{2}$$

where X_0 is the state vector; Y_0, Y_I are the vectors of the controlled and measured output, respectively; ξ is the sensor noise.

The matrices of Eqs. (2) have the following form

$X_0 = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$, $A_0 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}_{n\times n}$, $B_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b \end{bmatrix}_{n\times 1}$, $B_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{n\times 1}$, $C_0 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & 1 \end{bmatrix}_{n\times n}$, $C_I = [1 \quad 0 \quad \dots \quad 0]_{1\times n}$, $D_0 = [d]$,

where d is a parameter.

It is assumed that, in addition to estimating the state vector $\hat{X}_0 \approx X_0$, it is possible to obtain estimates of the total disturbance $\hat{f} \approx f$, then control actions can be formed as follows:

$$u = (u_0 - \hat{f})/b. \tag{3}$$

The control action term u_0 can be represented as

$$u_0 = K(R - \hat{X}), \tag{4}$$

where K is the $1 \times n$ gain matrix, selected in such a way as to satisfy a specified criterion for controlling the state vector of the nominal (compensated) plant

$$y^{(n)} = u_0.$$

In order to estimate the state vector X_0 and the disturbance f, the model of the plant should be augmented with a disturbance model. Reference [14] demonstrates that in the general case the disturbance model can be represented in the following form:

$$f(s) = 1/s^k, k \geq 1.$$

This model in the state space form is given as follows:

$$\dot{X}_f = A_f X_f, f = C_f X_f, \tag{5}$$

where $A_f = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}_{k\times k}$; $C_f = [0 \quad \cdots \quad 0 \quad 1]_{1\times k}$.

If a more accurate disturbance model is known, it can also be used. Augmenting representation (2) with model (5) and using the extended state vector $X_e = [X_0 \quad X_f]^T$, we obtain

$$\dot{X}_e = A_e X_e + B_e u, Y = C_e X_e + D_0 \xi, \tag{6}$$

where $A_e = \begin{bmatrix} A_0 & B_0 \ C_f \\ 0 & A_f \end{bmatrix}$; $B_e = \begin{bmatrix} B_0 \\ 0 \end{bmatrix}$; $C_e = [C_I \quad 0]$.

System (6) is observable if there are not any coincidence between the eigenvalues of the matrix A_f and the zeros of system (2). If this condition is met, estimates of the extended state vector can be obtained using the following observer:

$$\dot{\hat{X}}_e = A_e \hat{X}_e + B_e u + L\big(Y_I - C_e \hat{X}_e\big).$$

This equation can be given in standard state space form as

$$\dot{\hat{X}}_e = A_e \hat{X}_e + B_e u + B_u U_L,\ U_L = L\big(Y_I - C_e \hat{X}_e\big),$$

where B_u is the identity matrix with dimension $n + k \times n + k$.

Estimates of the state vector $X\hat{}$ and disturbance $f\hat{}$ can be defined as follows

$$\hat{X} = C_x \hat{X}_e, \quad C_x = [I \quad 0]_{n \times n+k},$$

$$f = C_F \hat{X}_e, \quad C_F = [0 \quad C_f]_{1 \times n+k},$$

where I is the identity matrix with dimension $n \times n$.

A block diagram of an equivalent representation of this dynamic system in the form of “nominal plant – ESO – inner loop controller (ELC)” is presented in Fig. 1. In this figure, the output vector components S_1, S_2 are the control and estimation errors of the extended state vector, respectively; Δ is the plant uncertainty caused by inaccurate knowledge of its parameters.

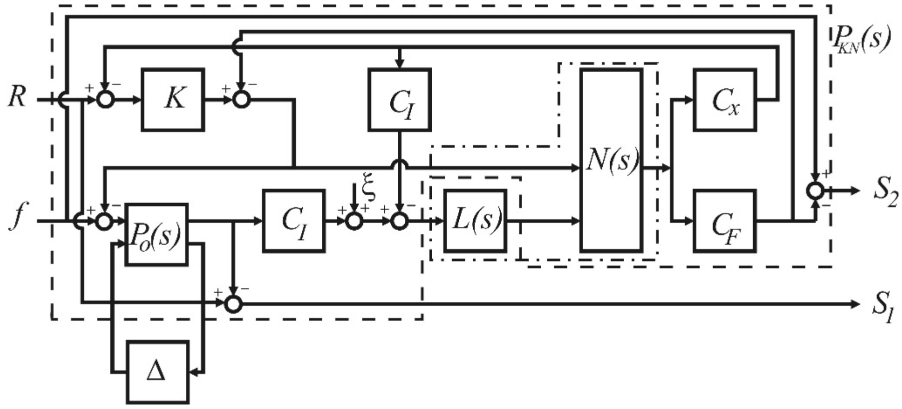

Fig. 1 – Block diagram of the equivalent system
“structurally perturbed nominal plant – ESO – ELC”

The ESO $L(s)$ is synthesized in such a way that the closed loop of the system satisfies the specified performance criteria, provided that the parameters of the plant vary within a given range. The closed loop design goals regarding the performance of control and state vector estimation are specified in the frequency domain. According to this approach, the system shown in Fig. 1 is augmented with two weighting functions W_{S1}, W_{S2} as follows:

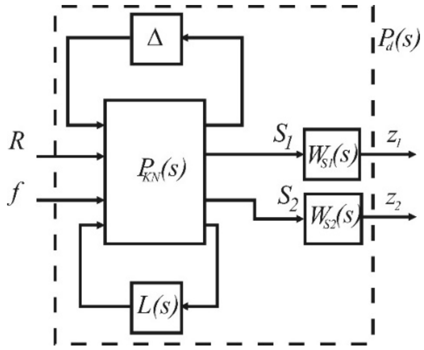

Fig. 2 – Block diagram of the augmented system

2. Robustness analysis. In Ref [13], a pretty flexible representation of uncertainty $\Delta(j\omega)$ was used to design a combined controller, which is suitable for accounting both structured disturbances (for example, caused by the deviation of the plant parameters the nominal ones) and non-structured disturbances. For this representation, robust stability is guaranteed for all $\Delta(j\omega)$, $\omega \in R$, when the following condition is satisfied

$$\bar{\sigma}\big(\Delta(j\omega)\big)\bar{\sigma}\big(H(j\omega)\big) < 1.$$

However, the perturbation $\Delta(j\omega)$, whose norm $\bar{\sigma}\big(\Delta(j\omega)\big)$ does not exceed $1/\bar{\sigma}\big(H(j\omega)\big)$, is completely unstructured. The consequence of this is a significant conservatism of estimates of stability regions in the case of structured disturbances, and in particular parametric uncertainties.

In the case of structured disturbances, it is a more efficient way to use another robustness measure, which uses the concept of structured singular values [15]. According to this approach, the perturbation Δ is represented in a block-diagonal form

$$\Delta = \begin{bmatrix} \Delta_1 & 0 & \cdots & 0 \\ 0 & \Delta_2 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \Delta_k \end{bmatrix}.$$

Each of the diagonal blocks Δ_i can have one of the following forms:

$\Delta_i = \delta$, where δ is a real number. In the case of representing the variation of a real parameter, the unit matrix I has a dimension of one;

$\Delta_i = \delta$, where δ is a stable matrix transfer function. This representation describes a scalar or repeating scalar dynamic disturbance;

Δ_i is a stable matrix transfer function. This representation describes a multidimensional dynamic disturbance.

The model of a dynamic system with this representation of uncertainty is shown in Fig. 3.

If M is a complex matrix of dimension $n \times m$ and a set D of structured perturbation matrices of dimension $m \times n$, then the structured singular value for M and D is the quantity $\mu(M)$, which is defined as follows

$$\frac{1}{\mu(M)} = \inf\bar{\sigma}(\Delta),\ \Delta \in D, d \quad (I - M \quad) = 0.$$

If $\det(I - M \quad) \neq 0$ for all $\Delta \in D$, then $\mu(M) = 0$.

The structured singular value $\mu(M)$ is the inverse norm of the smallest perturbation from the considered class D under that makes the matrix $I + M$ singular. It follows from this that the larger $\mu(M)$, the less perturbation Δ is necessary to make the matrix $I + M$ singular.

An alternative definition of $\mu(M)$ can be given as follows

$$\mu(M) = \max\rho(M \quad),\ \Delta \in D, \bar{\sigma}(\Delta) \leq 1,$$

where ρ is the amplitude of the largest eigenvalue.

Figure 4 shows a system consisting of block H (nominal plant and combined controller) and block Δ (structured disturbance). In this figure, the following notations are used: w is the controller input; z is the control error; p is the input for block Δ; q is the output for block Δ.

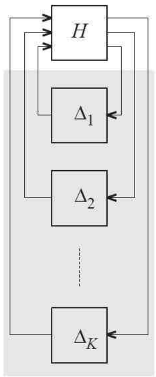

Fig. 3 – Plant with structured uncertainty

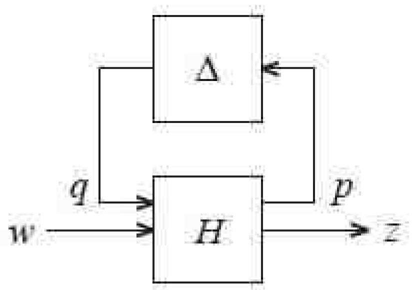

Fig. 4 – Block diagram for robust stability analysis

The disturbances are scaled in such a way that $\|\Delta\|_\infty \leq 1$. H^Δ denotes the transfer function from w to z. This function is scaled and satisfactory control performance is ensured when $\|H^\Delta\|_\infty \leq 1$.

Given this, the conditions for the system robustness with respect to structured disturbances can be formulated as follows: the system has robust stability and robust performance with respect to all structured disturbances Δ only if [15]

$$\mu(H) < 1 \ . \tag{7}$$

A linear-fractional transformation can be used to represent a parametric uncertainty of the plant. We consider a complex matrix M connecting the vectors r and v as follows

$$v = M \ \ .$$

Dividing r and v by upper and lower parts (Fig. 5a), this expression can be rewritten as follows

$$v_1 = M_1 \ r_1 + M_1 \ r_2,$$

$$v_2 = M_2 \ r_1 + M_2 \ r_2.$$

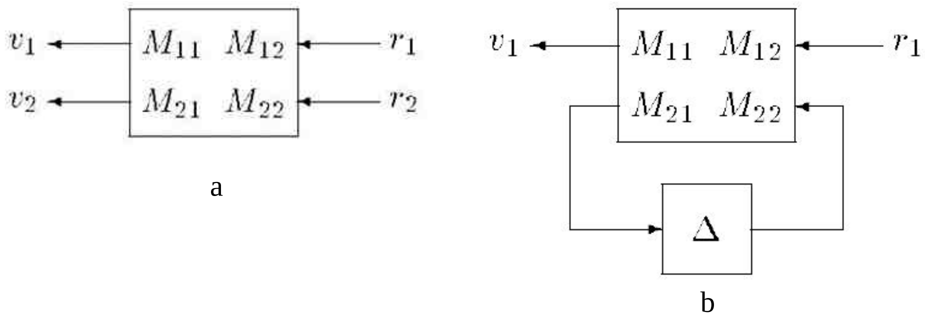

Fig. 5 – Interpretation of linear-fractional transformation

The matrix Δ establishes the relation between the vectors r_2 and v_2 as follows

$$r_2 = \Delta v_2,$$

then the relation between между r_1 and v_1 (рис. 5a) is given as

$$v_1 = [M_1 \ \ + M_1 \ \Delta(I - M_2 \ \Delta)^{-1} M_2 \ \ ] r_1 = F_L(M, \Delta) r_1.$$

Here the notation $F_L(M, \varDelta)$ means that the lower loop of M is closed by the matrix $\varDelta$ (Fig. 5b). Using a linear-fractional transformation, the uncertain parameters of the considered model $b = b_n + \partial b$, $d = d_n + \partial d$ are as follows

$$b = b_n + \partial \ \ \varDelta_1, \varDelta_1 \in [-1,1], d = d_n + \partial \ \ \varDelta_2, \varDelta_2 \in [-1,1],$$

$$b = F_L\left(\begin{bmatrix} b_n & \partial b \\ 1 & 0 \end{bmatrix}, \varDelta_1\right), d = F_L\left(\begin{bmatrix} b\ {}_n & \partial d \\ 1 & 0 \end{bmatrix}, \varDelta_2\right).$$

3. Extended state observer synthesis. To synthesize an extended state observer in accordance with criterion (7), the system, shown in Fig. 1, is represented in the following standard form:

$$\dot{X} = A \quad + B_1 F + B_2 U_L, \tag{8}$$

$$Z = C_1 X + D_1 \ \ F + D_1 \ \ U_L, \tag{9}$$

$$Y = C_2 X + D_2 \ \ F + D_2 \ \ U_L, \tag{10}$$

where X is the state vector; Z is the controlled output; Y is the measured output; F is the disturbance vector; U_L is the control actions.

The weight functions $W_{S1}(s), W_{S2}(s)$ have their corresponding representations in the state space

$$\dot{X}_i = A_i X_i + B_i U_i, i = S, T, \tag{11}$$

$$Y_i = C_i X_i + D_i U_i, j = 1,2. \tag{12}$$

Given this representation, as well as the features of the augmented system $P_d(s)$ (Fig. 2), the matrices and vectors included in (8) – (12) can be represented in the following form:

$$X = \begin{bmatrix} X_0 \\ \hat{X}_e \\ X_{S1} \\ X_{S2} \end{bmatrix}, A = \begin{bmatrix} A_0 & -B_0 \ (C_F + KC_x)/b_n & 0 & 0 \\ 0 & A_e - B_e(C_F + KC_x)/b_n & 0 & 0 \\ -B_{S1}C_0 & 0 & A_{S1} & 0 \\ 0 & -B_{S2}C_F & 0 & A_{S2} \end{bmatrix},$$

$$D_1 \ \ = \begin{bmatrix} D_{S1} & 0 \\ 0 & D_{S2} \end{bmatrix},$$

$$D_1 \ \ = [0], C_2 = [C_I \quad -C_I C_e \quad 0 \quad 0], D_2 \ \ = [d_n + \varDelta \ \ ], D_2 \ \ = [0], F = [R \quad f]^T,$$

$$B_1 = \begin{bmatrix} B_0 \ K/b_n & B_0 \\ B_e K/b_n & 0 \\ B_{S1} & 0 \\ 0 & B_{S2} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ B_u \\ 0 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} -D_{S1}C_0 & 0 & C_{S1} & 0 \\ 0 & -D_{S2}C_F & 0 & C_{S2} \end{bmatrix},$$

$$X_0 = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}, \qquad A_0 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}_{n\times n}, \qquad B_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_n + \Delta \end{bmatrix}_{n\times 1},$$

$$B_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{n\times 1}, C_0 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & 1 \end{bmatrix}_{n\times n}, A_f = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}_{k\times k},$$

$$C_I = [0 \quad \cdots \quad 0 \quad 1]_{1\times n}, C_f = [0 \quad \cdots \quad 0 \quad 1]_{1\times k}.$$

where b_n is the nominal value of parameter b; Δ is the maximum deviation of the value of parameter b from the nominal value.

The gain matrix of the ILC for is selected in the following form:

$$K = [\omega_r^2 \quad 2\omega_r],$$

where ω_r is the controller's bandwidth.

An approach based on the approximate calculation of the structured singular value is used to synthesize a robust combined controller. This approach uses the following property of the structured singular value [16]

$$\mu(M) \le \bar{\sigma}[D \quad \bar{D}^{-1}].$$

Using this property, the minimization task for

$$\mu_{P_L} = \sup_{\omega\in R} \mu\big(P_L(j\omega)\big)$$

is replaced by the problem of minimizing the following upper bound

$$\sup_{\omega\in R} \bar{\sigma}[D(j\omega)P_L(j\omega)\bar{D}^{-1}(j\omega)] = \|DP_L\bar{D}^{-1}\|_\infty, \tag{13}$$

where for each frequency ω of the diagonal matrices $D(j\omega)$ and $\bar{D}^{-1}(j\omega)$ is chosen in such a way as to obtain the minimum value of the upper bound. The task of minimizing the expression (13) with respect to observers L is a standard H_∞-optimization problem. This method is called D-K iterations [17]. The capabilities of this method are limited to the class of complex structured disturbances. Within this method, real disturbances caused by uncertain parameters can be approximated by dynamic uncertainties.

In the cases of mixed real and complex disturbances, the D-G-K iteration method [18] can be used. This method utilizes the following property. If a real number $\beta > 0$ and block diagonal matrices D and G exist that satisfy the condition

$$\bar{\sigma}\left((I + G^2)^{-\frac{1}{4}}\left(\frac{1}{\beta}D \quad \bar{D}^{-1} - j\omega\right)(I + G^2)^{-\frac{1}{4}}\right) \le 1, \tag{13}$$

then

$$\mu(M) \le \beta. \tag{14}$$

Using this methodology, the procedure for synthesizing the combined controller with an ESO taking into account the plant parametric uncertainty can be implemented using the following steps.

Step 1. Select a model of the nominal plant $P_0(s)$.

Step 2. Specify the structured uncertainty matrix Δ, taking into account the known ranges of parameter variations.

Step 3. Select the ILC gains K, based on the chosen criterion for controlling the state vector of the nominal plant $P_0(s)$.

Step 4. Select the order of the disturbance model (in cases where a more accurate disturbance model is known, it can also be used).

Step 5. Select weighting functions $W_{S1}(s)$, $W_{S2}(s)$ considering the requirements on state control, as well as the expected spectrum of disturbances and sensor noise.

Step 6. Calculate the representation matrices of the augmented system P_d.

Step 7. Find numerically matrices A_L, B_L, C_L, D_L, and μ_m by using the algorithm of D-G-L iterations.

Step 8. If μ_m does not satisfy condition (7), adjust the parameter ω_r of the ILC (D-G-L-K iteration) until the required value of μ is obtained.

Step 9. If it is not possible to obtain the required value of μ, the order of the disturbance model should be increased (step 4) and (or) the requirements for the combined control system should be relaxed by selecting other weighting functions (step 5).

4. Numerical example. For numerical calculations, the following matrices of the state space representation of the plant and disturbance model are used

$$A_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0 \\ b_n + \Delta \end{bmatrix}, C_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_i = [1 \quad 0], A_f = [0], C_f = [1],$$

where $b_n = 5000 \text{ kg} \cdot m^2$, $\Delta = \pm 1000 \text{ kg} \cdot m^2$.

The following first-order frequency filters are chosen as the weighting functions:

$$W_{S1} = \frac{0{,}8 \quad s+2}{s+0{,}0}, W_{S2} = \frac{0{,}7 \quad s+5}{s+0{,}0}.$$

Functions W_{S1} and W_{S2} set the minimum requirements for the upper bounds of the bandwidths of the system "nominal plant – ESO – ILC" for the reference and disturbance inputs at the level of 2 rad/s and 50 rad/s, respectively. The ESO is synthesized using the D-G-K iteration algorithm (13), (14). Table 1 summarizes the results of such a synthesis for a selected ILC bandwidth of $\omega_r = 10$ rad/s. As can be seen from this table, the best structured singular value is obtained on the 7th iteration, but it does not satisfy condition (7). Thus, for these initial data it is not possible to synthesize the ESO that ensures robust stability and performance of the closed loop with respect to the parametric uncertainty of the plant.

Table 1 – Optimization results for $\omega_r = 10$ rad/s.

Iteration	ESO order	Scaling order	$\|H\|_\infty$	Max μ
1	7	0	5.277	2.885
2	9	2	2.163	2.156
3	11	4	2.032	1.417
4	11	4	1.457	1.306
5	11	4	1.335	1.258
6	11	4	1.307	1.245
7	11	4	1.269	1.002
8	19	10	0.924	33.665

Next, we increase the upper bound of the controller bandwidth and repeat the D-G-K iterations for $\omega_r = 12$ rad/s. The results of such iterations are given in Table 1. As can be seen from this table, in this case, the ESO ensures robust stability and robust performance of the closed loop with respect to the parametric uncertainty of the plant, since the best singular value satisfies condition (7). Figures 6 – 13 show frequency variations of the structured singular values and the sensitivity functions of the obtained controller for each of these iterations.

Table 2 – Optimization results for $\omega_r = 12$ rad/s.

Iteration	ESO order	Scaling order	$\|H\|_\infty$	Max μ
1	7	0	6.155	2.926
2	9	2	1.878	1.396
3	11	4	1.388	1.280
4	11	4	1.268	0.996
5	19	10	1.013	0.996
6	17	8	0.923	0.996

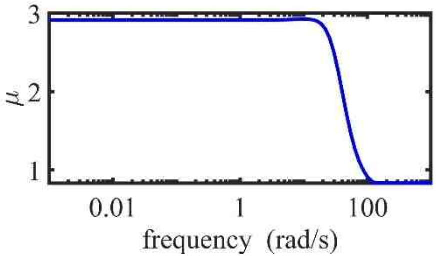

Fig. 6 – Variation of structured singular value (1-st iteration)

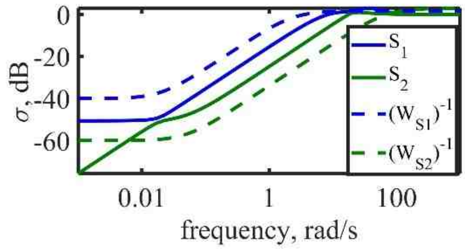

Fig. 7 – Variation of sensitivity function (1-st iteration)

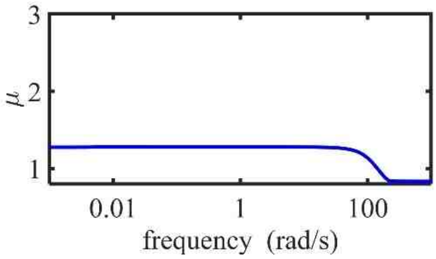

Fig. 8 – Variation of structured singular value (3-rd iteration)

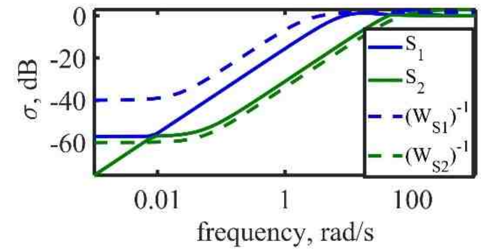

Fig. 9 – Variation of sensitivity function (3-rd iteration)

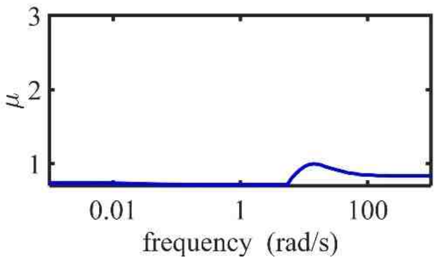

Fig. 10 – Variation of structured singular value (3-th iteration)

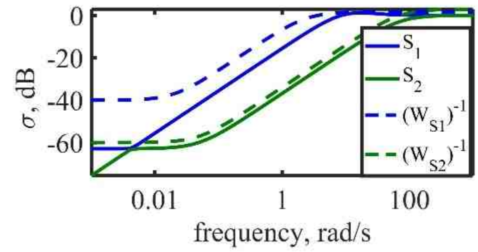

Fig. 11 – Variation of sensitivity function (3-th iteration)

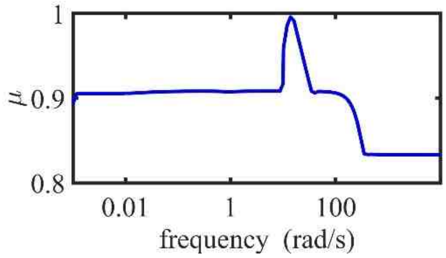

Fig. 12 – Variation of structured singular value (6-th iteration)

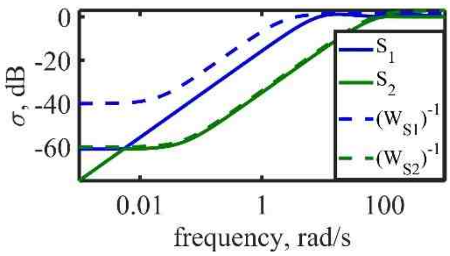

Fig. 13 – Variation of sensitivity function (6-th iteration)

Figures 14 – 16 show the step responses of the system for different values of the uncertain parameter b. As can be seen from these figures, the parameter variation in the given range does not affect the transient processes. This is additional confirmation of the robust performance of the combined controller.

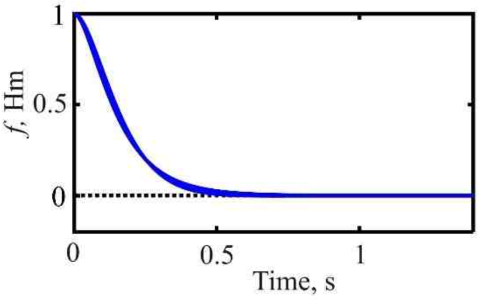

Fig. 14 – Step response
(input R – output S_1)

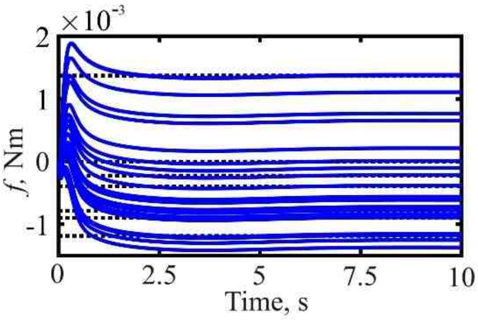

Fig. 15 – Step response
(input f – output S_1)

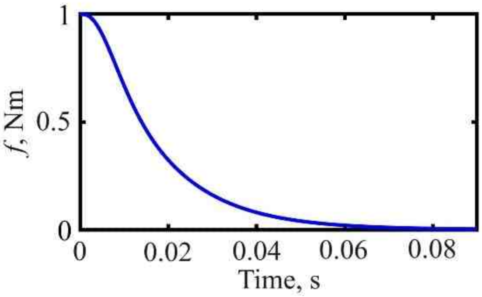

Fig. 16 – Step response
(input f – output S_2)

Table 3 presents the results of the synthesis of the ESO with an ILC bandwidth of $\omega_r = 15$ rad/s. Analysis of these results allows us to conclude that such a bandwidth does not ensure the stability of the closed loop against parametric uncertainty. Thus, ω_r should also be selected iteratively.

Table 3 – Optimization results for $\omega_r = 15$ rad/s.

Iteration	ESO order	Scaling order	$\|H\|_\infty$	Max μ
1	7	0	7.476	2.962
2	9	4	2.706	2.480
3	11	4	2.040	1.413
4	11	4	1.311	1.246
5	11	4	1.230	1.202
6	11	4	1.183	1.170
7	11	4	1.152	1.147
8	11	4	1.151	1.146

Conclusions. A procedure for synthesizing an extended state observer is proposed considering the closed loop of the combined control system. The methodology of optimization of structured singular values is used for accounting the plant parametric uncertainty, which is presented as a structured disturbance. For the synthesis of combined controllers, an algorithm of D-G-L-K iterations is proposed, the peculiarity of which is an additional adjustment of the gains of the inner loop controller after finding the optimal observer. Such adjustment allows obtaining such structured singular values that guarantee robust stability and performance of the dynamic system. This approach allows reducing the conservatism of the combined controller, but it leads to controllers of a high order due to the use of additional scaling.

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