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ON THE ISSUE OF CREATING SPACE BASED SHADING AND LIGHTING SYSTEMS FOR EARTH SURFACE

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Проблема забезпечення сприятливих кліматичних умов на певній території є глобальною для людства. В умовах глобальної зміни клімату вирішення цієї проблеми може стати ключовим для управління національними економіками багатьох країн. На теперішній момент було запропоновано ряд технічних пропозицій для створення засобів, які можуть дозволити досягти мети глобального контролю клімату. Ці рішення включають концепцію зміни орбіти Землі, використання аерозольних морських і стратосферних технологій і концепцію «космічних систем затінення». Однією із перспективних запропонованих концептуальних розробок є технологія «космічних систем затінення». Враховуючи значне наукове підґрунтя та схожий принцип дії освітлювальних (ілюмінаційних) систем, запропоновано використовувати їх одночасно з «космічними системами затінення».

Отже, метою роботи є розробка структури математичної моделі балістико-навігаційного забезпечення космічної системи затінення та освітлення (КСЗО). Для досягнення мети дослідження визначено структурні модулі КСЗО, які включають космічну індустріальну платформу для виробництва модулів КСЗО, пасивні і активні космічні модулі затінення та освітлення, службові космічні апарати. Визначено узагальнені конструктивні схеми модуля затінення та освітлення. На основі особливостей структурних модулів КСЗО розроблено структуру математичної моделі балістико-навігаційного забезпечення КСЗО. Структура балістико-навігаційного забезпечення складається з п'яти структурних елементів: орбітальний рух, кутовий рух навколо центра мас, система керування супутниковою орієнтацією і орбітою, оптичний та геодезичний модулі. Комплексне використання цих модулів може дозволити вирішити низку конкретних завдань, пов'язаних з вибором проєктних параметрів КСЗО на етапі концептуального проєктування. Перелік цих завдань було обґрунтовано та визначено для подальших досліджень.

Ключові слова: *космічна система затінення та освітлення, балістичне та навігаційне забезпечення, структура математичної моделі, концептуальне проєктування, космічна система затінення.*

The problem of ensuring favorable climatic conditions in a particular territory is global for mankind. In the context of the global climate change, its resolution may be crucial for national economy management in many countries. By now, a number of engineering solutions have been proposed to develop means that may allow one to achieve the goal of global climate control. These solutions include an Earth orbit change concept, aerosol marine and stratospheric technologies, and a "sunshade" concept. One of the promising conceptual developments is a space-based "sunshade" technology. Taking into account the significant scientific background and a similar principle of operation of lighting (illuminative) systems, they were suggested to be used simultaneously with "sunshade" systems.

The goal of this work is the development of a structure for a mathematical model of the ballistics and navigation of a space-based shading and lighting system (SBSLS). To do this, SBSLS structural modules were identified: a space-based industrial platform for SBSLS module production, passive and active shading and lighting modules, and service spacecraft. Generalized construction arrangements of the shading and lighting module were decided on. Based on the features of the SBSLS structural modules, a structure for a mathematical model of SBSLS ballistic and navigational support was developed. The structure comprises five components: an orbit estimator, an attitude motion estimator, an attitude and orbit control system, an optical estimator, and a geodetic estimator. A number of specific problems involving the choice of SBSLS design parameters at the conceptual design stage were identified and justified for further investigation. The combined use of the above modules may allow one to solve them.

Keywords: *space-based shading and lighting system, ballistics and navigation support, structure of mathematical model, conceptual design, space-based sunshade system.*

Introduction. The problem of global climate-control means creation became significant task for mankind last decades. From one hand it is connected with the hypothesis of global warming, from another this means are needed for the countries which territories is located in "non-convenient" climate zones. These "non-convenient" zones, for example, can be equatorial, subequatorial or tropical with high percent of deserts (Saudi Arabia, UAE, Amman, Izrael, Jordan, etc.), polar and subpolar zones of such countries as USA, Canada, Finland, etc. (Fig. 1).

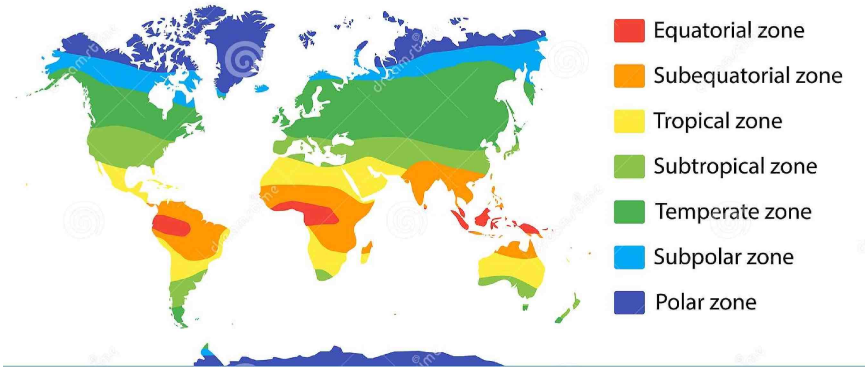


Fig. 1 – Geographical climate zones

Thus, these climate condition in some periods of the year can impact negatively on some parts of national economies of these countries such as agriculture, tourism, international trade, food production, etc.

Taking it into account the problem of creation of climate-control means can be devoted in two subproblems:

1) creation of global means of Earth protection from solar radiation in the frame of decreasing global warming process (in the frame of global warming concept);

2) creation regional and local means for time-periodic climate change in determined territory of the Earth.

Literature review and problem statement. So, in the frames of global warming theory, some concepts of Earth protection from the solar radiation impact were developed. One of these concepts is based on the Earth orbit change [1, 2]. The one of well noted scientist of last decades Robert Zubrin estimated that for increasing Earth distance out from Sun by $5\ \%$ it is needed the acceleration of $3.8\cdot 10^{-14}\ \frac{\text{m}}{\text{s}^2}$ [1]. Taking into account the value of Earth mass, R. Zubrin calculated that to achieve this acceleration it would be needed the force (thrust) of about 227 billion newtons. In turn, further it was determined, that thruster to generate the force of such value needs a lot of energy (about $4.7\cdot 10^{35}$ J) [2]. Taking these aspects into account, there is no possibility and technologies to realize this idea today. That's why it can be only long-term perspective for mankind.

The next approach of Earth protection from increasing the intensity of solar radiation is based on stratospheric aerosol geoengineering [3 – 6]. So, the aerosol injection in stratosphere using tethered balloons was described in the paper [3]. The authors carried out a deep analysis of the means of delivering aerosols to altitudes up to 20 km in this paper. The usage of naval artillery, different types of missiles, railguns, coilguns, aircrafts, rigid towers, single-use balloons, retrievable balloons and tethered balloons was compared in the paper. So, using several criteria of comparison (time at altitude, minimum development time, environmental impact and social impact) it was justified the advantage of the tether balloons creation relative to others means. However, the influence of aerosols on the ecology of the atmosphere has not been carried out. In turn, the new design of using "Sail aircraft" for aerosol delivering was proposed in the paper [4]. Using previous experience of creation airplanes with high altitude of flight (RB-57F and U-2S) it was developed a new airplane which can carry about 13600 kg of aerosol. The planned flight altitude where aerosol injection is started in this case is 20 km. It was determined that the approximate cost of the mission using “Sail aircraft” would be between \$1.1 and \$1.5B, (USD FY2020). In turn, taking into account the instability of oil market, the fuel prices can increase which will leads to significantly increasing of mission costs. Also, the airplane is the additional source of atmospheric pollution which can make its use unattractive from an environmental point of view.

Taking into account difficulties of aerosol impact on the stratosphere chemical properties the authors of paper [5] talked about necessity of the significant number of experiments. These experiments are needed for clarifying or confirmation/rejection of present simulation results of geoengineering researches. Also, the scenarios of stratospheric aerosol injection deployment which was being analyzed by researchers in article [6], showed a great cost (cost-per-deployed-tone of aerosol is 2400\$ at an altitude of 20 km) of decreasing the averaged temperature of the Earth by $2\,^{o}C$. In turn, it was determined that if this technology would be used only for subpolar and polar regions it can allow to reduce costs by 3 times. However, decreasing of the temperature by $2\,^{o}C$ can’t significantly help planet. This can only partially slow down the process of melting glaciers in the Arctic and Antarctica, which, in turn, will reduce the pace of rise of the world ocean.

Thus, considering the information from overview which related the stratospheric aerosol geoengineering technologies it can be concluded [3–6]:

1) the technology has significant theoretical background;

2) experiments are being carried out in this area, and a significant part of them are planned;

3) today there is no clear understanding of which means of aerosol delivery are most advantageous to use, the answer is controversial;

4) the full degree of the presented aerosols impact on the ecology of the atmosphere (including air, precipitation, etc.) in the long term is not determined;

5) the technology hasn't operative control of Sun rays protection: there is no possibility of operative shading and after that lighting (if there is a necessity) of the territory;

6) the system has a significant inertia.

Thus, given the properties of these systems, they convenient can be used for tasks of decreasing global warming process and difficult be used for regional climate-control with time-periodic shades.

The next concept of Earth climate control is based on the marine cloud brightening [7 – 9]. It is the international collaboration of atmospheric scientists and other experts to advance understanding of cloud responses to aerosol particles [7]. The theoretical background of this concept is based on the ship tracks studies and their impact on cloud radiative properties. This study allowed to determine the aerosol–cloud radiative interactions and its influence on Earth albedo [8, 9]. It is assumed that the creation of such clouds will enable a better reflection of the sun's rays and reduce the rate of global warming. However, this concept requires the presence of significant water resources, the delivery of which to certain regions of the Earth can be difficult.

The name of the next concept is "Sunshade" [10 – 12]. It is based on usage special space shading technologies to reduce the impact of solar radiation on the Earth. So, the adventures of perspectives of space sunshades creation are described in the paper [10]. The author substantiated the need of such systems creation in the context of geoengineering climate control tools development. The economic justification of this concept has been analyzed, but the technical implementation wasn't described. The technology for implementing the "sunshade" system, which is based on the deployment of a large circular disk near the Sun-Earth Lagrangian equilibrium point L1, is presented in [11]. The paper emphasizes the advantage of the location of the disk around the L1 point from the point of view of ensuring the balance of the forces acting on this disk. So, the position is determined by the condition of compensation of the solar pressure and Earth's gravity forces by the gravitational force of the Sun. It was determined that in order to reduce the radiation intensity by 1.7%, the radius of the "sunshade" disk should be 915 km. In turn, it was calculated that using the disc with a radius of 1434 km can nullify the impact of greenhouse gas emissions worldwide [11]. However, creating and launching a disc with such mass-dimensional parameters at point L1 is quite a difficult task and requires a lot of energy.

Later research works in this scientific area show the need to create a constellation of "sunshade" spacecraft [12]. It was estimated that for reducing the amount of sunlight for the Earth by 1 %, a solar "shield" with an area of $3.79 \cdot 10^{12}$ m^2 would be needed [12]. Considering the proposed area of 9000 m^2 for one sail, it was calculated that from $4.2 \cdot 10^8$ to $1.5 \cdot 10^9$ space sail vehicles located in the vicinity of the point L1 would be required to achieve the necessary area of the solar "shield". With the existing modern technologies for launching satellites into orbits, this will require from 330000 to 830000 launches. A rather significant range of these estimates is explained by the use of different types of sail configurations, especially the difference in the Q parameter (sail efficiency factor) [12]. It is planned that the launches will be carried out in orbits close to equatorial, with an altitude of 2000 km. After that, the sailing spacecraft, using the thrust of the solar radiation pressure have to perform orbit transition to the last destination near the vicinity of the L1 point. According to estimates, the orbital transition time in this case will take 1096 days [12].

However, taking into account the given general models of the functioning of "sunshades" systems [10 – 12], a complete mathematical model of such systems functioning was not presented. Also, these mathematical models given in works [11, 12] describe only a special case of peculiarities of "sunshade" modules operation near point L1. In turn, the full range of possible orbits for "sunshade" modules location hasn't been analyzed and described.

In turn, the analysis of the previous researches [1 – 12] show the actuality of global climate-control systems creation. One of the perspective directions of such system creation is based on the development of "sunshade" space-based systems [9 – 12]. In turn, the estimates of the source consumption which is required for creation such system make this concept doubtful from the point of practical realization both at present time and in near future. However, with the development of a new concept of space industrialization [13 – 14], the cost of creating such systems can be reduced by using on-orbit producing technology. This economic effect can be explained by the absence of need for multiple launches into orbits of "sun-shade" modules which requires a lot of costs. On the other hand, the use only "sunshade" technologies for reducing solar rays impact to the Earth doesn't solve the problem of global climate control completely. This can be explained by the presence of both global warming and cooling processes, which can periodically replace each other.

Taking this into account, it is advisable to include both "sunshade" and lighting (illuminative) modules of the Earth's surface as the one of the modules in the space-based global climate control system. In turn, the lighting (illuminative) technologies has a great scientific background since project “Znamya” until modern formation flight satellites including “writing sunlight” technologies [15 – 16].

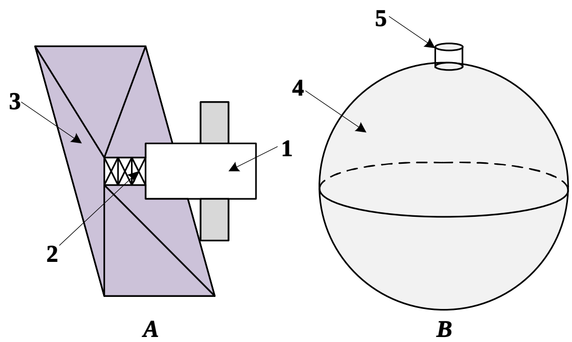
Taking this into account, the **aim of the work** is the development of the mathematical model structure of ballistic-navigational support for space-based shading and lighting system. Taking into account the purpose of the article, the following tasks has been formed:

1. Determination of the structural components of the space-based shading and lighting system.
2. Development of the mathematical model structure of space-based shading and lighting system (SBSLS) ballistic and navigation support.
3. Determination of further research tasks using the developed SBSLS ballistic and navigational mathematical model structure.

Determination of the structural components of the space-based climate-control system. Analyzing previous researches of the development of “sunshade” systems [10 – 12] it can be seen, that only one type of sail geometry and principle of launch were proposed in the frames of this concept (one or a lot of flat sails elements which are launched from Earth). However, it can be other approaches of “sunshade” or lighting modules types and creation. The use of such approaches and the development of new types of SBSLS can significantly increase their efficiency. Thus, the new structure of SBSLS is proposed by authors, including approaches of SBSLS modules in-space producing using industrial platform concept [13 – 14]. It is assumed that the production of SBSLS in space can expand the possibilities for creating new types of such systems, the production of which was previously not available (or difficult available) on Earth conditions. Taking this hypothesis into account it can be determined next structural modules of SBSLS:

1) Space-based industrial platform for SBSLS modules production;
2) “Sunshade” modules for reducing solar rays impact;
3) Illuminative mirror modules for increasing solar rays impact;
4) Service spacecrafts of SBSLS.

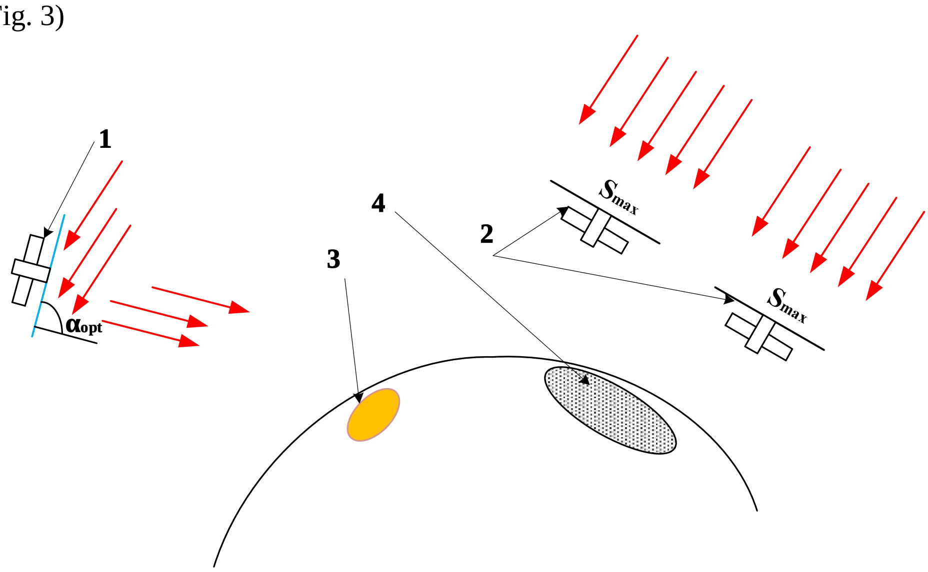
Considering this new approach, it is assumed that service spacecraft will be execute a delivering function for SBSLS modules from the space industrial platform to target orbits. The delivering process will include two phases - orbital transition from space industrial platform orbit to target orbit and deploying of "sunshade"/illuminative orbital groups on target orbit. Taking it into account, there are two general types of “sunshade” and illuminative modules of SBSLS can be distinguished (Fig. 2). These types are flat sail active module (Fig. 2, A) and volume inflatable passive module (Fig. 2, B).



1 – Service spacecraft of SBSLS; 2 – Mast element of the sail; 3 – Flat sails "Sunshade"/Illuminative element; 4 – Volume passive "Sunshade"/Illuminative element; 5 – Compression valve for pressurization

Fig. 2 – Main types of "Sunshade"/Illuminative modules of space-based climate-control systems

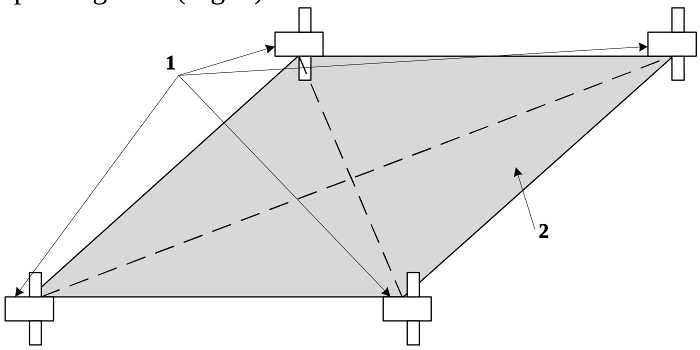
So, the first type of "sunshade" and illuminative modules which is based on the use of flat sail elements require attitude and angular control. It is can be explained by the necessity of providing angular stabilization of flat sail element with the respect to the directive vector of solar radiation flux. The goal of such orientation and stabilization support is providing maximal "sunshade" area in the case of shading and optimal reflection angle of solar rays in the case of illumination (Fig. 3)



1 – Service illuminative spacecraft of SBSLS; 2 – Service shading spacecraft of SBSLS; 3 – Illuminative region of the Earth; 4 – Shading region of the Earth; α_{opt} – Optimal value of reflection angle for illuminative modules of SBSLS; $S_{\max}$ – maximal value of the sail surface area projection on the plane which is orthogonal to sun flux vector (condition of maximal shading)

Fig. 3 – Special Sun orientation of illuminative and "sunshade" modules of SBSLS

However, using separate spacecrafts with sail elements require a lot of launches of such satellites [12] or a great number of their on-orbit producing in the case of creation on-orbit industrial plants [13, 14]. Taking it into account there is a task of determination of the optimal ratio of service spacecrafts number to the sail area. One of the directions in this task solving is based on the determination of the required number of service spacecrafts which are needed for providing stabile motion of the super-large sail (Fig. 4).



1 – Number of service spacecrafts which provides super-sail controllable and stabile flight; 2 – Super-sail shading or illuminative element

Fig. 4 – Special Sun orientation of illuminative and “sunshade” modules of SBSLS

Considering the construction presented in figure 3 it can be seen that the control system of sail attitude motion will be distributed. Taking it into account the control algorithm more rational to synthase using mobile control methods [17] and sliding control approaches [18]. Thus, it can be an additional direction in the development of these control methods, which can be interested for both theoretical and applied engineering aspects of science.

The second type of “sunshade”/illuminate modules is based on the use of passive inflatable volume space structures which don’t require providing stability and control. So, the main task in this case is to determine and develop the most optimal body geometry of illuminative modules which allow providing sun rays reflection to the Earth surface during non-oriented free motion. In turn, for “sunshade” modules the most optimal body geometry can be sphere, spheroid and ellipsoid. The advantage of using passive modules is absence of on-board energy consumption and propulsion. However, in this case, a number of other problems arise that need to be solved for ensuring the normal functioning of such systems. Among these tasks are the following:

– determination of the orbits for passive “sunshade”/illuminate modules location which will satisfice the requirements of shading/illuminating process periodicity for selected territory;

– development of the special service spacecrafts which provide delivery and deployment of the SBSLS passive modules groups on selected orbits;

– providing deorbiting of the SBSLS passive modules groups after the finish of the mission in the case of their location on Low Earth Orbits (LEO) or transporting to disposal orbits with further utilization in the case of location higher than region LEO.

In turn, for the determination of the “sunshade”/illuminate modules parameters including geometry, orbital position and number of units it is necessary to develop the mathematical model of SBSLS functioning. The main stage of this mathematical model synthesis is the development of the SBSLS ballistic and navigation support structure.

Development of the mathematical model structure of space-based climate-control system ballistic and navigation support. Ballistic and navigational support (BNS) is needed for the determination of SBSLS orbital, relative and attitude motion, estimation of geodetic parameters of Earth lighting and shading zones and synthesis of control algorithm for SBSLS modules orbital constellations (groups). Structurally ballistic and navigation model can be divided to the next parts:

1) Orbit estimator – the structural part of BNS mathematical model which is needed for SBSLS modules orbital motion analysis including relative motion of SBSLS modules formation flying.

2) Attitude motion estimator – the structural part of BNS mathematical model which is needed for the analysis of SBSLS modules motion around their centers of mass (COM).

3) Attitude orbital control system (AOCS) – the structural part of BNS mathematical model which is needed for determination and synthesis of control laws of SBSLS attitude motion.

4) Optical estimator – the structural part of BNS mathematical model which is needed for the determination of illuminating or shading sail/volume element areas.

5) Geodetic estimator – the structural part of BNS mathematical model which is needed for determination current position of illuminating or "sunshade" module position with the respect to the Earth territories and geodetic parameters of lighting and shading areas on the Earth surface.

Taking into account proposed structural components of the BNS it will be useful to use reference frames which was fully described in works [19 – 22]. These reference frames are: J2000 reference frame, WGS-84, Orbital general reference frame (OGRF), Local vertical local horizontal (LVLH) [22] reference frame, STW reference frame [20], Body reference frame (BRF) which is connected with *i*-th SBSLS module [19]. Using these reference frames, it is advisable to represent the structure of the mathematical model as a sequence of algorithmic blocks with logical and mathematical connections. Considering this it is very convenient to use table form with order of blocks (Table 1).

Table 1. Structure of ballistic and navigational support

Structural part	**Reference frames**	**General mathematical description**
1	2	3
Orbit estimator	J2000, WGS-84, OGRF, STW	**Orbital motion [20]** $$\left.\begin{aligned} \frac{dh}{dt} &= \frac{h^2}{\xi}\cdot T \\ \frac{de_x}{dt} &= h\cdot\left[S\cdot\sin F + T\cdot\left[(\xi+1)\cdot\cos F + e_x\right] - W\cdot e_y\,\frac{\eta}{\xi}\right] \\ \frac{de_y}{dt} &= h\cdot\left[-S\cdot\cos F + T\cdot\left[(\xi+1)\cdot\sin F + e_y\right] + W\cdot e_x\,\frac{\eta}{\xi}\right] \\ \frac{di_x}{dt} &= \frac{h\cdot\tilde{\varphi}}{2\xi}W\cdot\cos F \\ \frac{di_y}{dt} &= \frac{h\cdot\tilde{\varphi}}{2\xi}W\cdot\sin F \\ \frac{dF}{dt} &= \frac{\xi^2}{h^3\mu} + W\cdot h\cdot\eta \end{aligned}\right\}$$

1	2	3
Orbit estimator	J2000, WGS-84, OGRF, STW	**Determining cartesian cinematic parameters of SBSLS modules motion [19]** $\mathbf{r_{sh}} = \begin{bmatrix} X_{sh} \\ Y_{sh} \\ Z_{sh} \end{bmatrix} = \begin{pmatrix} \begin{pmatrix} \cos\Omega\cos\omega - \\ -\sin\Omega\sin\omega\cos i \end{pmatrix} & \begin{pmatrix} \sin\Omega\cos\omega + \\ +\cos\Omega\sin\omega\cos i \end{pmatrix} & \sin i \sin\omega \\ \begin{pmatrix} -\cos\Omega\sin\omega - \\ -\sin\Omega\cos\omega\cos i \end{pmatrix} & \begin{pmatrix} -\sin\Omega\sin\omega + \\ +\cos\Omega\cos\omega\cos i \end{pmatrix} & \sin i \cos\omega \\ \sin\Omega\sin i & -\cos\Omega\sin i & \cos i \end{pmatrix}^{\mathrm{T}} \sqcup$ $\sqcup \begin{bmatrix} r_{sh}\cos(u-\omega) \\ r_{sh}\sin(u-\omega) \\ 0 \end{bmatrix},$ $\mathbf{V_{sh}} = \begin{bmatrix} V_{x.sh} \\ V_{y.sh} \\ V_{z.sh} \end{bmatrix} = \begin{pmatrix} \begin{pmatrix} \cos\Omega\cos u - \\ -\sin\Omega\sin u\cos i \end{pmatrix} & \begin{pmatrix} \sin\Omega\cos u + \\ +\cos\Omega\sin u\cos i \end{pmatrix} & \sin i \sin u \\ \begin{pmatrix} -\cos\Omega\sin u - \\ -\sin\Omega\cos u\cos i \end{pmatrix} & \begin{pmatrix} -\sin\Omega\sin u + \\ +\cos\Omega\cos u\cos i \end{pmatrix} & \sin i \cos u \\ \sin\Omega\sin i & -\cos\Omega\sin i & \cos i \end{pmatrix}^{\mathrm{T}} \sqcup$ $\sqcup \begin{bmatrix} V_{pr.sh} \\ V_{pr.sh} \\ 0 \end{bmatrix},$ $\mathbf{dR}_{kj} = \left[X_{sh.k} - X_{sc.j} \quad Y_{sh.k} - Y_{sc.j} \quad Z_{sh.k} - Z_{sh.j} \right]^{\mathrm{T}},$ $\mathbf{dV}_{kj} = \left[V_{x.sh.k} - V_{x.sc.j} \quad V_{y.sh.k} - V_{y.sc.j} \quad V_{z.sh.k} - V_{z.sh.j} \right]^{\mathrm{T}},$
Attitude motion estimator	J2000, WGS-84, OGRF, STW, LVLH	$J\frac{d\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times (J\boldsymbol{\omega}) = \mathbf{M^c} + \mathbf{M^p},$ $\frac{d\boldsymbol{q}}{dt} = \frac{1}{2}\boldsymbol{q} \circ \mu K$
AOCS	J2000, WGS-84, OGRF, STW, LVLH	$M^c_{pr.x} = -\left(J_{xx} + J_{xy} + J_{xz}\right)\left(K_1\Delta\omega_x + K_2\Delta q_x + K_3\int\Delta q_x dt\right),$ $M^c_{pr.y} = -\left(J_{yx} + J_{yy} + J_{yz}\right)\left(K_1\Delta\omega_y + K_2\Delta q_y + K_3\int\Delta q_y dt\right),$ $M^c_{pr.z} = -\left(J_{zx} + J_{zy} + J_{zz}\right)\left(K_1\Delta\omega_z + K_2\Delta q_z + K_3\int\Delta q_z dt\right).$ Expected service spacecraft equipment Actuators of service spacecraft: magnetorquers, reaction wheels. Sensors of service spacecraft: magnetometer, star tracker, solar sensors, gyroscopes. Noise and Kalman filter model will depend on equipment characteristics

1	2	3
Optical estimator	J2000, WGS-84, OGRF	**Current orbit position relative to the Sun [20]** $\mathbf{R}_{sun} = \left[\cos L_{\square} \quad \sin L_{\square} \cos\varepsilon \quad \sin L_{\square} \sin\varepsilon\right]^{T}$, $\beta = \arcsin\left(\frac{R_E}{r_{sh}}\right)$, $\begin{cases} \text{if} \quad \arccos\left(-\mathbf{R}_{sun} \cdot \mathbf{r_{sh}}\right) \le \beta & \text{eclipse part of orbit} \\ \text{if} \quad \arccos\left(-\mathbf{R}_{sun} \cdot \mathbf{r_{sh}}\right) > \beta & \text{solar part of orbit} \end{cases}$, **Shading estimation [11]** $l_{sh} = R_{\square} \frac{d_{sh}}{d_{\square}} \sqrt{\frac{\Delta S}{S}}$, Estimation of shading position $\mathbf{ort}^{wgs}_{sun \to sc} = \left[r^{wgs}_{sun \to sh.x} \quad r^{wgs}_{sun \to sh.y} \quad r^{wgs}_{sun \to sh.z}\right]^{T}$, $Tar\_line = \begin{cases} R^{wgs}_{sh.x} + r^{wgs}_{sun \to sh.x}\lambda, \\ R^{wgs}_{sh.y} + r^{wgs}_{sun \to sh.y}\lambda, \\ R^{wgs}_{sh.z} + r^{wgs}_{sun \to sh.z}\lambda, \end{cases}$, Intersection with Earth ellipsoid by solving equation for λ [23]: $X_{el} = R^{wgs}_{sh.x} + r^{wgs}_{sun \to sh.x}\lambda$, $Y_{el} = R^{wgs}_{sh.y} + r^{wgs}_{sun \to sh.y}\lambda$, $Z_{el} = R^{wgs}_{sh.z} + r^{wgs}_{sun \to sh.z}\lambda$, $\frac{X_{el}^2}{a_{el}^2} + \frac{Y_{el}^2}{a_{el}^2} + \frac{Z_{el}^2}{b_{el}^2} = 1.$ **Lighting estimation [16]** $m = -2.5 \cdot \log\left(\frac{I}{I_{ref}}\right)$ $I = \frac{S \cdot A_r \cdot \rho \cdot \tau \cdot \cos(\gamma)\sin(\theta_{si})}{4d_{si}^2 \tan^2(\gamma_{beam})}$
Geodetic estimator	WGS-84	Using iterative algorithm [23] for determination geodetic latitude, longitude and altitude of illuminative and shading modules

The formulas in Table 1 denoted: $e_x = e \cdot \cos(\omega + \Omega)$; $e_y = e \cdot \sin(\omega + \Omega)$; $i_x = \tan\left(\frac{i}{2}\right) \cdot \cos\Omega$; $i_y = \tan\left(\frac{i}{2}\right) \cdot \sin\Omega$; $h = \sqrt{\frac{p}{\mu}}$; $\xi = 1 + e_x \cos F + e_y \sin F$; $\eta = i_x \sin F - i_y \cos F$; $\tilde{\varphi} = 1 + i_x^2 + i_y^2$; $F = \omega + \Omega + \vartheta$; e is the eccentricity of the "sunshade"/illuminative i-th module orbit; i is the inclination of the "sunshade"/illuminative i-th module orbit; Ω is the RAAN of the "sunshade"/illuminative i-th module orbit; ω is the argument of perigee of "sun-shade"/illuminative i-th module orbit; p is the focal parameter of "sun-shade"/illuminative i-th module orbit; μ is the gravitational constant; r_{sh} is the module of radius vector of "sunshade"/illuminative i-th module, $r_{sh} = \dfrac{a\left(1-e^2\right)}{1+e\cos\vartheta}$; a is the semi-major axis of "sunshade"/illuminative i-th module orbit, $a = p/\left(1-e^2\right)$; ϑ is the truth anomaly of "sunshade"/illuminative i-th module; S, T, W are the radial, transversal and normal projections of perturbative accelerations in STW reference frame; $\mathbf{r_{sh}} = \left[X_{sh} \quad Y_{sh} \quad Z_{sh}\right]^{\mathrm{T}}$ is the radius vector of current "sunshade"/illuminative i-th module position in J2000 reference frame; $\mathbf{V_{sh}} = \left[V_{x.sh} \quad V_{y.sh} \quad V_{z.sh}\right]^{\mathrm{T}}$ is the velocity vector of current "sun-shade"/illuminative i-th module position in J2000 reference frame; $V_{pr.x} = \sqrt{\dfrac{\mu}{p}} \cdot e \cdot \sin\left(u-\omega\right)$; $V_{pr.y} = \sqrt{\dfrac{\mu}{p}} \cdot \left(1 + e \cdot \cos\left(u-\omega\right)\right)$; $u = \omega + \vartheta$ is argument of orbit latitude; $\mathbf{dR}_{kj} = \left[X_{sh.k} - X_{sc.j} \quad Y_{sh.k} - Y_{sc.j} \quad Z_{sh.k} - Z_{sh.j}\right]^{\mathrm{T}}$ is the relative position vector of "sunshade"/illuminative k-th module relative to j-th module in "sunshade"/illuminative orbital group; $\mathbf{dV}_{kj} = \left[V_{x.sh.k} - V_{x.sc.j} \quad V_{y.sh.k} - V_{y.sc.j} \quad V_{z.sh.k} - V_{z.sh.j}\right]^{\mathrm{T}}$ is the relative velocity vector of "sunshade"/illuminative k-th module relative to j-th module in "sunshade"/illuminative orbital group; $J = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{bmatrix}$ is tensor of inertia of "sunshade"/illuminative i-th module or service spacecraft with respective components; $\boldsymbol{\omega} = \left[\omega_x \quad \omega_y \quad \omega_z\right]^{\mathrm{T}}$ is the vector of angular velocity of "sun-shade"/illuminative i-th module or service spacecraft in BRF coordinates; $\mathbf{M^c} = \left[M_x^c \quad M_y^c \quad M_z^c\right]^{\mathrm{T}}$ is the control torque of active flat "sun-shade"/illuminative i-th module or service spacecraft in BRF coordinates; $\mathbf{M^p} = \left[M_x^p \quad M_y^p \quad M_z^p\right]^{\mathrm{T}}$ is the total perturbative torque of "sun-shade"/illuminative i-th module or service spacecraft in BRF coordinates; $\boldsymbol{q} = \left[q_0 \quad q_x \quad q_y \quad q_z\right]^{\mathrm{T}}$ is the quaternion of rotation from LVLH reference frame to BRF in BRF coordinates; $M_{pr.x}^c$, $M_{pr.y}^c$, $M_{pr.z}^c$ are the program torques components which are estimated for the service spacecraft or active SBSLS modules attitude motion control in BRF reference frame; K_1, K_2, K_3 are gains of controller; $\Delta\omega_x$, $\Delta\omega_y$, $\Delta\omega_z$ are misalignments by angular velocities in each control channel; Δq_x, Δq_y, Δq_z are misalignments by quaternion in each control channel; $\mathbf{R}_{sun}$ is the direction unit vector from Earth to Sun in J2000 reference frame; $L_{\odot}$ is the Sun's ecliptic longitude; ε is obliquity of the ecliptic; β is the angle that defines the entry and exit positions of eclipse/solar parts of the orbit; l_{sh} is the characteristic size of *i*-th "sunshade" module; d_{sh} is the distance from *i*-th "sunshade" module to Earth surface (current altitude of the orbit); $d_{\odot}$ is the distance from Sun to Earth; $R_{\odot}$ is the radius of Sun; $S = 1367$ W / m^2 is the intensity of solar energy flux; ΔS is the amount by which the solar energy flux intensity is reduced due to shading by *i*-th "sunshade" module; $\mathbf{ort}_{sun\to sc}^{wgs} = \begin{bmatrix} r_{sun\to sh.x}^{wgs} & r_{sun\to sh.y}^{wgs} & r_{sun\to sh.z}^{wgs} \end{bmatrix}^{\mathrm{T}}$ is the unit vector of the direction from Sun to the i-th "sunshade" module and it's components in WGS-84 reference frame; $R_{sh.x}^{wgs}$, $R_{sh.y}^{wgs}$, $R_{sh.z}^{wgs}$ are the coordinates of the i-th "sunshade" module current position in WGS-84 reference frame; Tar_line is the target line from Sun to the i-th "sunshade" module; λ is the non-dimensionless parameter; m is the pixel magnitude of reflected light intensity; I is the reflected light intensity at the observed locality; $I_{ref} = 2.56 \cdot 10^{-6}$ lux is the reference intensity [16]; A_r is the solar reflector area of *i*-th illuminative module; ρ is the reflectivity coefficient; γ is the incident angle of solar rays; θ_{si} is the elevation angle of the *i*-th illuminative module estimated at target lighting region central point; d_{si} is the distance between *i*-th illuminative module reflector and target lighting region central point; τ is the atmospheric transmissivity [16].

Thus, the structure of the BNS mathematical model of SBSLS has been formalized. Using this structure will clarify the peculiarities of SBSLS modules orbiting during the development of full mathematical model and software for SBSLS functioning simulation.

Determination of further research tasks using the developed SBSLS ballistic and navigational mathematical model structure. The next step of SBSLS study will be connected with conceptual design of "sunshade" and illuminative modules [24]. It is stage correspond to phase A (Conceptual phase) and B (Phase of satellite components requirements determination of the development process phases for different spacecrafts [25]. Using developed SBSLS ballistic and navigation support structure it can be started two phases of software development for study "sunshade"/illuminative modules orbiting: "algorithm in the loop" [25] and "software in the loop" [25]. During these phases it will be necessary to develop software which can allow to simulate the SBSLS modules flight more closely to real orbiting in space. After that, it is arising a number of pre-project tasks which are connected with the SBSLS modules orbits selection. Among these tasks are the following:

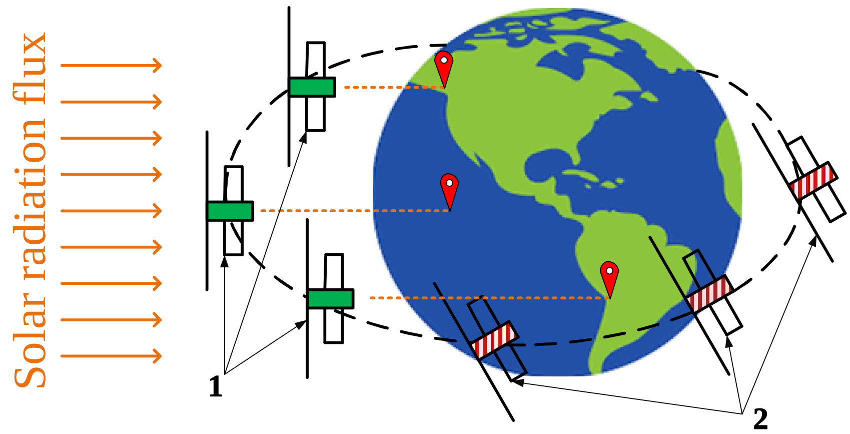
1) determination of the "sunshade" and illuminative modules target orbits;

2) determination a number of "sunshade" and illuminative modules in each target orbit;

3) determination of the transition orbits for service spacecrafts;

4) determination deorbiting process of "sunshade" and illuminative modules at the end of lifetime.

During solving the first and second tasks of SBSLS modules orbit selection it should be taken into account the "sunshade" or illuminative orbital group efficiency. This efficiency can be determined as a relation of the number of current activated SBSLS modules to the total number of SBSLS modules on selected orbit (Fig. 5).



1 – activated SBSLS modules; 2 – inactivated SBSLS modules

Fig. 5 – SBSLS orbital group efficiency

So, the activated SBSLS modules will be referred to as modules which can perform "sunshade" or illuminative tasks at current time. For example, for “sunshade” part of SBSS it will be modules which are located in solar part of the orbit and reduce solar radiation flux intensity above Earth surface (Fig. 5). The determination of the current active / inactive status of such SBSLS module can be estimated by solving intersection equation in “optical estimator” (Table 1) with further analysis of geodetic anchoring using “geodetic estimator” (Fig. 5). In turn, using “orbit estimator” there is a task of SBSLS modules activating/inactivating periodicity determination and a number of activated SBSLS modules in each time period. Thus, it is possible to determine the effectiveness of any SBSLS constellation at current time in a certain orbit.

The third task which is connected with orbit transition determination for service spacecraft includes next subtasks:

– determination of attitude controllability of service spacecraft in each work mode using “attitude motion estimator” and “AOCS” (Table 1);

– gains selection for attitude motion controller using “AOCS” (Table 1);

– calculation of orbit transition maneuver (space industrial platform $\rightarrow$ “sunshade”/illuminative target orbit) for service spacecrafts using “orbital estimator”;

– determination of program attitude spacecraft orientation in the process of passive “sunshade”/illuminative modules separation.

Also, in addition to the above, the tasks of AOCS development may arise when controlling active “sunshade” or illuminative modules. In this case AOCS should provide necessary stabilization of flat sail in Sun tracking mode during lighting or shading mission.

The fourth task which is related to deorbiting process analysis should be solved in satisfaction to the requirements of satellites lifetime restriction [26]. Thus, solving this list of problems, it will allow to select design parameters of SBSLS system at the conceptual stages of designing

Conclusion. The paper analyzes the features of creating various systems of global climate control. It has been determined that one of the promising areas in the creation of modules for such systems is based on developing of "sunshade" space systems. Taking into account a significant scientific background and similar principle of operation of illuminative systems it has been proposed to use them simultaneously with “sunshade” systems. It is considered that such approach can expand the efficiency of space-based systems of solar radiation flux control in the frame of global climate control system.

One of the general tasks for providing normal functioning of space systems is creation ballistic and navigational support for them. Considering this, it has been developed the structure of mathematical model for ballistic and navigational support for space-based shading and lighting system. This structure consists of five parts using whish allow to solve a lot of ballistic and navigational tasks including orbiting estimation, attitude motion estimation, AOCS development, navigational positioning relative to the Sun and Earth. So, using this ballistic and navigational support it will be possible to determine design parameters of SBSLS AOCS system, orbit parameters, positioning and initial data for creation propulsion, thermal and electrical system of SBSLS modules.

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