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DEPLOYMENT OF THE SPACE TETHER IN THE CENTRIFUGAL FORCE FIELD WITH ALIGNMENT TO THE LOCAL VERTICAL

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Об'єктом дослідження є мала орбітальна зв'язка двох тіл, яка повинна бути розгорнута з космічного апарата таким чином, щоб після завершення розгортання вона була розташована вздовж місцевої вертикалі. Тіла зв'язки вважаються однаковими по масі; нитка, що з'єднує тіла, вважається невагомою. Мета проведеного дослідження полягає в тому, щоб побудувати програмний закон управління довжиною зв'язки, що враховує зміну кінетичного моменту зв'язки під впливом гравітаційного моменту від центрального ньютонівського поля сил. Проведено дослідження процесу розгортання космічної зв'язки двох тіл у полі відцентрових сил з вирівнюванням зв'язки наприкінці розгортання вздовж місцевої вертикалі. Для розгортання використовується попередня закрутка зв'язки довкола бінормалі до орбіти. Дослідження складається із двох етапів. У першому етапі проводиться побудова закону управління довжиною зв'язки, який забезпечує виконання запланованого розгортання. На цьому етапі використовуються рівняння руху зв'язки, записані у сферичних координатах для часткового випадку руху зв'язки у площині орбіти. На другому етапі дослідження проводиться чисельне моделювання динаміки розгортання зв'язки під дією побудованого програмного закону управління довжиною зв'язки. Як математична модель зв'язки використовуються рівняння Hill–Clohessy–Wiltshire, що описують просторовий рух тіл. Ці рівняння не містять у явному вигляді довжину зв'язки як змінну. Тому рівняння модифікуються. Зусилля натягу зв'язки, що входить у ці рівняння, виражено через програмний закон зміни довжини та двох його перших похідних за часом. Новизна проведеного дослідження полягає у побудові програмного закону управління, що дозволяє розгорнути зв'язку вздовж місцевої вертикалі за одну стадію. При виконанні дослідження використовувалися методи аналітичної механіки, чисельні методи і методи, розроблені авторами. Отримані результати дозволяють встановити області значень параметрів розгортання, що дозволяють проводити розгортання такого типу. Проведено оцінку помилок чисельного моделювання. Практична значущість отриманих результатів полягає у можливості проводити розгортання малих зв'язок на орбіті з вирівнюванням їх наприкінці режиму вздовж місцевої вертикалі за одну стадію з управлінням по довжині зв'язки без необхідності проводити надалі гасіння лібраційних коливань.

Ключові слова: *управління, космічна зв'язка, розгортання, місцева вертикаль, одна стадія.*

This study is concerned with a small orbital tether of two bodies to be deployed from a spacecraft so that upon completion of the deployment it turns out to be aligned along the local vertical. The bodies of the tether have equal masses, and the thread connecting the bodies is supposed to be massless. The objective of the study is to build a program law of tether length control taking into account the variation of the angular momentum of the tether under the action of the gravitational torque from the central Newtonian field of forces. The deployment mode of the space tether in a centrifugal force field with its alignment at the conclusion of the deployment along the local vertical is studied. To produce centrifugal forces, the tether is pre-spinned about the orbit binormal. The study consists of two steps. The first step involves the construction of a tether length control law that would provide the planned deployment. At this step, use is made of the tether motion equations written in spherical coordinates for the special case of the tether motion in the orbital plane. A numerical simulation of the tether deployment dynamics is carried out at the second step using the constructed program law of tether length control. Hill-Clohessy-Wiltshire's equations are used as a mathematical model of the tether. They describe the spatial motion of the tether bodies. These equations do not contain the tether length as a variable in explicit form. Therefore, these equations are modified. The tether tension force appearing in these equations is expressed in terms of the program law of tether length change and its two first time derivatives. The novelty of the study consists in the construction of a program control law that allows the tether to be deployed along the local vertical in a single stage. The study used methods of analytical mechanics, numerical methods, and methods developed by the authors. The obtained results make it possible to find the ranges of values of the deployment system parameters allowing a deployment of this type. The errors of the numerical simulation are estimated. The practical significance of the obtained results consists in the possibility of deploying small tethers in orbit with their alignment at the conclusion of the deployment along the local vertical in a single stage with controlling the tether length without the need for further dumping of libratory oscillations.

Keywords: *Control, space tether; deployment, local vertical, one stage*

Introduction. The ideas of use of the bodies connected by long flexible thread in space go back to K. E. Tsiolkovsky's works and have more than century history. The novelty and the originality of the problems and research techniques of the dynamics of the space tethered systems (STS) attract to them attention of the experts. Since the beginning of the 80th, the STS area is forming to the separate area of the space research.

Among the problems connected with creation of the STS, the specific place is held by the problem of deployment of the tether of two space objects in the specified state. The large number of publications is devoted to this problem. Their distinctions are defined first of all by the accepted tether physical model and also by the nature of the control applied to deployment of the tether in its final state.

In practice of application of the STS, their most demanded configuration is such at which the center of gravitation of the system moves in a circular orbit and the tether is aligned along the local vertical. Its vertical configuration has a stable relative equilibrium in the orbital frame of reference in case of the constant length of the tether. The majority of practical applications is connected with use of the radial tether of two bodies – the spacecraft and subsatellite. So, for example, high stability of a radial STS made it possible to suggest its use as the basic bearing element for various options of solar space power plants [8, 9], for gravity-gradient stabilization of spacecraft and for transportation of cargoes between modules (space elevator) [12], as the lunar space elevator [10, 17] and the space "escalator" [8]. Microgravity conditions on end bodies of the tether can be used both for scientific and technological processes, and for life support of spacecraft and space stations [13, 16], for example, for pumping of liquid [10], or for improvement of living conditions in orbit [5].

The configuration along the local vertical loses the stability at change of the tether length in accordance with the theorem of change of the angular momentum [14]. The important operational requirement that the subsatellite at deployment or partial retrieval must remain on the local vertical, can be reached only at a special control. In principle, for the motion control of the tether end bodies the traction devices located directly on them can be used, however it complicates significantly all system. Therefore, there were many publications, whose results show possibilities of easier ways of control by the tether state.

Most publications devoted to research related to the deployment of tethers consider their motion in circular orbits. At the same time, studies appear that consider the motion of the tether in elliptical orbits [21].

The modes of deployment of orbital tethered systems are known providing release of the thread with regulation of its release velocity. The description and analysis of various modes of deployment of orbital tethered systems with control of the deployment velocity are provided in works [6, 11, 12,15].

The ways of deployment of STS are known providing release of the thread with its tension control. The description of such modes of deployment and devices for their realization is provided in works [18 – 20].

At present, the large amount of the works is known describing various ways of deployment when the tether came to alignment along the local vertical at the end of deployment [4, 8, 11, 12, 23].

Analyzing the known publications, one may come to the conclusion that the overwhelming majority of them that are devoted to such a deployment of tethers describe two-stage process of deployment. At the first stage, the bodies of a future tether (or the body of one subsatellite) most often are put to separate orbits by method of spring pushing away. Then it is necessary to execute suppression of libratory oacillations, which inevitably arise at the end of the first stage.

Depending on the method used for calm of the libratory oscillations, this process can demand either the large number of time, or special instrumental equipment of the maternal spacecraft. This publication is intended for creation of such a mode of the tether deployment, which can deploy the tether for one stage into the state aligned along the local vertical.

There are several works devoted to single-stage deployment of tethers in the centrifugal force field, but not in the state aligned along the local vertical, and in the state of rotation either with the prespecified angular velocity, or with the prespecified length [2, 3]. Besides, in these works the action of the planet gravitational field was not taken into account, and the tension control of the deployment was supposed by means of Coulomb forces and viscous friction forces for which it is difficult to sustain the adequate accuracy of their realization.

Though contours of the tethers having the "astronomical" sizes are already looked through in the theoretical researches, the main attention of the researchers is still directed to small tethers study. As a rule, it is the so-called small space tethers having lengths from several dozen meters to several kilometers. Such tethers are of practical interest as they are suitable both for the solution of independent scientific tasks, and for improvement, justification and check of the number of hypotheses concerning the dynamics of deployment of tethers and ways of their stationary motion creation. The tether with extent about tens meters can serve as a standard of length for calibration of both onboard, and ground-based optical and radar systems, as integral sensors of a planet force field, atmospheric probe, etc. It is expedient to carry out tethers up to several kilometers long, in particular, in electrodynamic option to use for correction of spacecraft orbits or deorbiting of the space systems, which fulfilled their term.

The present publication is devoted to study of the deployment dynamics of two bodies from spacecraft with the objective of their alignment along the local vertical at the end of deployment, using only one stage. The tether can remain connected with the spacecraft in two mass or in one mass options after completion of the deployment, or be separated from it with transition to some orbit, close to the initial one.

Physical model. Consider the case of the tether deployment directly from a spacecraft. Let the mass of the spacecraft be significantly greater than the total mass of the tether. Consider a case when the tether with length of several dozen meters must be deployed. To use the known two stage modes for this purpose is problematic since the tether tension at such sizes of the tether and mass of the end bodies about (1–10) kg is very weak and can be lost that leads to loss of the tether controllability. Therefore the decision was made to study the opportunity to use centrifugal forces for deployment of the tether. Creation of centrifugal forces requires preliminary rotation of the tether in its initial state with further increase the tether length according to the found law, which will provide the tether of the set length at the end of deployment. Even if creation of the one branch tether on a spacecraft is required, it is expedient to deploy two identical opposite branches in the case under consideration. Otherwise, one rotating branch can create significant perturbations of the spacecraft attitude as it is problematic to create rotation of the tether in the orbital plane passing through the spacecraft mass center.

It is possible to create initial rotation of the tether, which is deployed from a spacecraft in different ways. For example, two spring pushers can be installed on a spacecraft. The pushers push away tether bodies at the command from the Earth or from the on-board computer so that moving away the tether begins to rotate about a normal to the orbital plane.

One can also use microengines of small propulsion on end bodies of the tether or on the rotating beam from whose ends tether bodies separate at the initial instant.

Advantage of spring pushers is that they more precisely create identical impulses of repulsive forces for end bodies in the directions set in the orbital frame. Their shortcoming is that the impulse of the angular momentum created by them at pushing away of the tether bodies has to be compensated by the spacecraft attitude control system. At the same time, pair of microengines on the bodies of the tether will not bring perturbations in orientation of the spacecraft. The moment of pulling forces created by them is not put to the spacecraft and does not break its attitude. But at the same time, there is the problem of exact orientation of micro-engines in the orbital frame of reference. Without focusing further on these issues, consider that at the initial instant of the deployment process both bodies are identical in size, have the opposite in direction velocities and are in the orbital frame at equal distances from the rotation center. Such a tether is attached to the spacecraft by the center of masses (but not the center of gravitation) at the time of the deployment.

Completely deployed tether has the angular momentum, the vector of which is directed along the binormal to the spacecraft orbit, and its magnitude is equal [8]

$$\left|\vec{K}^C(t)\right| = 2\, m\, r^2 \mu^{or},$$

where m is mass of one end body, r is half of length of the tether L, μ^{or} is angular velocity of the orbital spacecraft motion.

The problem of the control system of the tether deployment is to reach by the tether such a value of its angular momentum during deployment when it coincides with the local vertical. Such a value is formed at the expense of the initial angular momentum of the tether and influence of the gravitational torque at its deployment.

Mathematical model. Without loss in generality of the problem statement, choose the model of the tether as two equal point masses connected by a massless thread in a circular orbit with the radius 7,000 km in the field of attraction of the Earth. The neglectof the sizes of end bodies is proved by the fact that the modes of motion, in which winding of a tether on the end bodies is possible, are not considered further. The neglect of mass of a thread is justified for nonconducting electric current threads made of modern light materials. The total mass of such threads in the conditions of their weak tension in the Earth orbits is the insignificant part of the total tether mass. Besides, experience of previous researches, in particular, the analysis of results, which was carried out in [4], shows that the controlled motion of continual systems with the massive thread described by differential equations in partial derivatives practically does not differ from motion of the tether, whose mass is concentrated in end bodies.

Introduce the following right coordinate frame of reference: absolute frame – the axis $O_E Z_A$ coincides with the axis of the world and is directed to the Pole star, the axis $O_E Y_A$ is directed at the point of vernal equinox, the axis $O_E Y_A$ supple-ments the frame to right orthogonal, and orbital frame $Cx^{or} y^{or} z^{or}$ (Fig. 1) in which the axis Cx^{or} coincides with the local vertical, the axis Cy^{or} coincides with the velocity vector of the spacecraft mass center, the axis Cz^{or} coincides with the vector of the orbital angular velocity. For convenience of the description of motion of the tether end bodies, combine origin of the orbital frame of reference not with the mass center of the spacecraft, but with the mass center of the tether. At the same time, consider that the directions of its axes are parallel to the axes of the orbital frame with origin at the spacecraft mass center. Such a shift of the orbital frame is admissible, as the spacecraft sizes are negligible in comparison with the radius of its orbit.

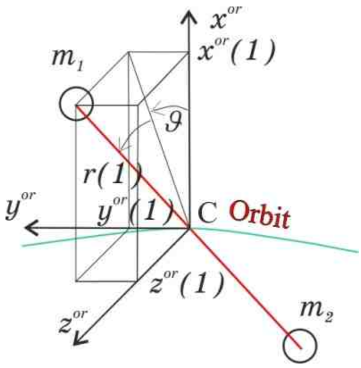

Fig. 1

Choose the central Newtonian field of forces as model of the gravitational field acting on the tether. The radius-vectors $\vec{r}_1, \vec{r}_2$ of the tether end bodies can be determined by their projections to the axes of the orbital frame: $\vec{r}_1 = \{x_1^{or}, y_1^{or}, z_1^{or}\}$, $\vec{r}_2 = \{x_2^{or}, y_2^{or}, z_2^{or}\}$. Choose these projections and their time derivatives as phase variables of the problem.

To investigate the possibility of realization of the considered method of deployment of the tether, it is necessary to construct the control law of the deployment mode in the beginning, and then to carry out the numerical simulation of this mode using the found control law.

It is necessary to carry out the error evaluation of solution in parallel with the numerical simulation. It will allow to judge the reliability of the obtained results.

Use the motion equations of the tether of variable length in the spherical coordinates [8] for creation of the program control law for the studied mode of deployment. They take the form at the used simplifying assumptions and in the notations accepted here:

$$\begin{aligned}
&\ddot{\gamma} + 2(\dot{\gamma} + \mu^{or})(\dot{r}/r - \dot{\vartheta}\tan\vartheta) + 3(\mu^{or})^2 \sin\gamma\cos\gamma = 0;\\
&\ddot{\vartheta} + 2\dot{r}/r\,\dot{\vartheta} + [(\mu^{or} + \dot{\gamma})^2 + 3(\mu^{or})^2\cos^2\gamma]\sin\vartheta\cos\vartheta = 0;\\
&\ddot{r} - r[\dot{\vartheta}^2 + (\mu^{or} + \dot{\gamma})^2\cos^2\vartheta + (\mu^{or})^2(3\cos^2\vartheta\sin^2\vartheta - 1)] + T/m = 0.
\end{aligned} \tag{1}$$

Here μ^{or} is the orbital angular velocity of the spacecraft, T is the tension force, m is mass of one end body. Other notations are clear from Fig. 1.

As the program motion of the tether is supposed to be created in the orbit plane, consider program value of the angle $\vartheta \equiv 0$. As a result, the system of the equations (1) reduces to the equations

$$\ddot{\gamma} + 2(\dot{\gamma} + \mu^{or})\dot{r}/r + 3(\mu^{or})^2 \sin\gamma\cos\gamma = 0; \tag{2}$$

$$\ddot{r} - r[(\mu^{or} + \dot{\gamma})^2 - (\mu^{or})^2] + T/m = 0. \tag{3}$$

The motion equations of the tether of variable length in the form of the Hill–Clohessy–Wiltshire (HCW) equations [8] are used farther for the numerical simulation of the mode of the tether deployment under control of the found program law. These equations describe the spatial motion of the end bodies concerning the mass center of the tether.

Following the traditional derivation of these equations, at the chosen direction of the orbital frame axes, they can be written down as follows:

$$\begin{aligned}\ddot{\vec{r}}_i = \{ & 2\mu^{or}\dot{y}_i^{or} + 3(\mu^{or})^2 x_i^{or} - T\, e_{ri}(1)/m_i,\\ & -2\mu^{or}\dot{x}_i^{or} - T\, e_{ri}(2)/m_i,\\ & -(\mu^{or})^2 z_i^{or} - T\, e_{ri}(3)/m_i\}, (i = 1, 2).\end{aligned} \tag{4}$$

It is necessary to know expression for the tether tension force T in each instant of time to close the system of twelve differential equations of the first order. This problem will be considered below.

The mistakes related to errors of calculations or to mistakes in mathematical computation of the researcher may become apparent in process of integration of the motion equations. Sometimes such mistakes slightly distort results in the considered range of parameters of the studied system and are imperceptible at first sight. To avoid such a situation, it is expedient to use independent check of the results, using the general theorems of the mechanics. For space objects of the considered type, which is subject to action of the known external forces, it is expedient to use the theorem of change of the angular momentum of the system for check of the integration results. Write down this theorem in the following integral form:

$$\vec{\mathrm{K}}^C(t) = \vec{\mathrm{K}}^C(t_0) + \int_{t_0}^{t} \vec{\mathrm{m}}^C({}^2)d{}^2 . \tag{5}$$

Here C is the point about which the angular momentum is calculated, $\vec{\mathrm{m}}^C$ is the external torque.

This theorem is satisfied both when the point C is fixed in the inertial frame, and in the case of the motion of the point C in the Keplerian orbit.

Now, knowing the law of change of the gravitational torque value during the tether deployment and integrating it in the real time, one may determine the current value of the angular momentum of the tether at any instant of time. On the other hand, the same value can be simply calculated, using the values of the phase variables of the problem in equations (4) obtained at solving of the initial value problem for equations (4). Comparison of the values of the angular momentum of the tether obtained by two various ways allows to draw the conclusion on reliability of the obtained results.

Scenario of deployment of the tether. Certain problems can arise for the control system of the spacecraft at using initial rotation of the tether for creation of centrifugal forces. If to deploy the tether with one body from the spacecraft, whose mass is significantly greater than the mass of the end body, then the rather large torque concerning its mass center will act on the spacecraft in the mode of initial rotation of the tether. The vector of this torque will rotate in the orbital plane, gradually decreasing with increasing in the tether length. It can cause disorientation of the spacecraft. The problem disappears, if the spacecraft has rather large mass and geometrical characteristics, or when two identical branch of the tethers are deployed at the same time in opposite directions. Consider further that one of these options is implemented. Study mainly the dynamics of deployment only of one branch of the tether.

Without dwelling on detail of constructive realization of the deployment device, consider that the rotation axis of the tether is fixed in the spacecraft frame of reference, passes through its mass center and is stabilized in the orbital frame during deployment. The deployment device creates initial rotation of the tether end bodies with the radius r_0 and the angular velocity $\dot{\vartheta}_0$. The end body m_1 (see Fig. 1) separates from the deployment device at $\vartheta = \vartheta_0$. At the same time the thread of the tether is deployed under the program law $r = r(t)$ from the device located on the spacecraft and rotates synchronously with rotation of the tether in order to the thread does not become twisted.

The tether appears aligned along the local vertical at the end of deployment. It can remain in such a state for execution of the tasks assigned to it.The scenario is also possible when the tether has to move away from the spacecraft in one or other direction in the close orbit. In that case, if the tether consists of one branch (i.e. has only one end body), it is possible to attach the second body on the free end of the tether and to separate it from the spacecraft. In that case the gravity centre of the tether passes to other orbit and begins to move away from the spacecraft with the velocity

$$V = R_{CT}(\mu_{CT}^{or})^2 - R_C(\mu_C^{or})^2 .$$

Here $\mu_C^{or} = GM / R_C^3$, $\mu_{CT}^{or} = GM / R_{CT}^3$, R_C is the radius of the spacecraft orbit, $R_{CT} \approx R_C \pm L/2$ is the new radius of the orbit of the tether (depending on the tether is deployed either in the nadir, or in zenith), $GM == 3.986004418*10^{14}\ m^3 s^{-2}$. So, for example, the tether of 5 km long shifted on the half of its length along the local vertical from the spacecraft, will move away in one or other direction along the tangent to the orbit with the velocity about 6 mm a second.

If the tether is deployed from the spacecraft has two branches, one can use the methods discribed in [1, 23] for its transfer to other orbit. At first one can retrieve completely one branch, and then deploy the second branch to the set length, without violating its alignment along the local vertical at the end.

Creation of program control. Consider further the algorithm of creation of the control of the deployment process of the one branch tether from the spacecraft in the centrifugal force field.

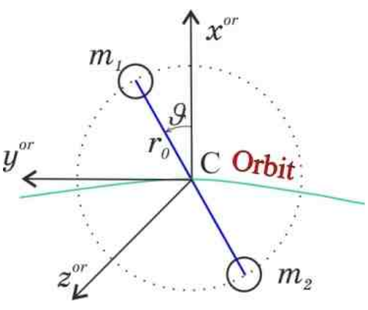

Fig. 2

The tether is installed in the beginning in the device of deployment, which rotates about the axis Cz^{or} with the set angular velocity (Fig. 2). Designate it as $\dot{\vartheta}_0$.

The body of the tether separates from the deployment device in the initial instant at value $\vartheta(0) = \vartheta_0$, supply of the thread in the branch of the tether begins and length of the branch begins to increase. During rotation, the periodic gravitational torque, whose maximum value increases in the magnitude with increasing the tether in length begins notably to act on the branch.

As through each tether turn its length monotonously increases, on each turn the angular momentum of the tether receives some increment.

When the angular momentum of the tether becomes equal to the value, which corresponds to the angular momentum of the tether of the current length aligned along the local vertical, supply of the thread terminates and the tether stops in the required position.

It is possible to determine values of key parameters defining the condition of the tether at the beginning and at the end of deployment on the basis of the described scenario of this mode. It is obvious that based on physical reasons, it is possible to set the following conditions:

$$r(0) = r_0; \quad r(T_f) = r_f; \tag{6}$$

The first condition is defined directly by the deployment device design. The second condition can be used further for determination in advance of the unknown time T_f of the deployment termination or for the choice of other parameters.

Two more conditions can be obtained from the condition of constancy of the tether length, both in the initial instant, and at the time of the deployment termination:

$$\dot{r}(0) = 0; \quad \dot{r}(T_f) = 0. \tag{7}$$

The following two conditions characterize lack of jump of the tension force, both at the beginning of deployment of the thread from the container, and at the time of the deployment termination:

$$\ddot{r}(0) = 0; \quad \ddot{r}(T_f) = 0. \tag{8}$$

It follows directly from equation (3), which has to be carried out during deployment at any instant.

Besides, the following boundary conditions of the mode for the angle g and its derivatives are obvious:

$$g(0) = g_0; \quad g(T_f) = 2n\text{®}. \tag{9}$$

As there are no obvious bases to accept the initial value of the pitch angle g other than zero, write down these conditions in the form

$$g(0) = 0; \quad g(T_f) = 2n\text{®} . \tag{10}$$

Further, also on the basis of the physical meaning, it is possible to write down two couples more of the boundary conditions:

$$\dot{g}(0) = \dot{g}_0; \quad \dot{g}(T_f) = 0. \tag{11}$$

$$\ddot{g}(0) = 0; \quad \ddot{g}(T_f) = 0. \tag{12}$$

Taking into account the equation (2), it is visible that the part of conditions (6)–(12) duplicate each other. On the other hand, conditions (7) and (8) on the basis of the equation (2) and conditions (10) can be replaced with two conditions

$$\dddot{g}(0) = -3(\mu^{or})^2\dot{g}_0; \quad \dddot{g}(T_f) = 0 . \tag{13}$$

Transform the conditions imposed on r to additional conditions on γ using equations (2), (3). Obtain ordinary differential equation of the first order with variable coefficients and with the corresponding initial condition proceeding from equation (2):

$$\dot{r} = -r\frac{3\left((\mu^{or})^2 \sin 2\gamma + 2\ddot{\gamma}\right)}{4\left(\mu^{or} + \dot{\gamma}\right)}, \ r(0) = r_0, \tag{14}$$

from which follows

$$r(t) = r_0 \exp\left[-\int_{T_2}^{t}\left(\frac{3(\mu^{or})^2 \sin(2\gamma(\tau)) + 2\ddot{\gamma}(\tau)}{4\left(\mu^{or} + \dot{\gamma}(\tau)\right)}\right)d\tau\right]. \tag{15}$$

It is possible to draw the conclusion from the given dependences that at the known law of change $\gamma(t)$ the law of change of the length $r(t)$ can be considered as function of the angle $\gamma(t)$, its derivatives, duration of deployment T_f and initial length of the tether. Thus, knowing the suitable law of change of the pitch angle of the tether vs, time, which corresponds to smooth approach of the tether to the local vertical, corresponds to the set final length of the tether, does not lead to loss of controllability of the deployed tether because of loss of its tension on some intervals of time, as a result of the solution of the initial value problem (8) the law of change of length of the tether can be found, which provides the solution of the formulated problem.

The law of change $\gamma(t)$ can be introduced in the form of any finite functional series, whose coefficients can be found from eight conditions (9), (10), (12), (13). For example, introduce the law of change of the pitch angle $\gamma(t)$ in the form of the power series

$$\gamma(t) = \sum_{i=0}^{7} c_i \left(\frac{t}{T_f}\right)^i. \tag{16}$$

Its coefficients found by using conditions (9), (10), (12), (13) is written as

$$\begin{gathered}
c_3 = -(\mu^{or})^2\dot{\gamma}_0/2; \ c_0 = 0; \quad c_1 = \dot{\gamma}_0; \quad c_2 = 0; \\
c_4 = (70\, nrot\, \pi - 20\dot{\gamma}_0\, T_f + (\mu^{or})^2\dot{\gamma}_0 T_f^3)/T_f^4\ ; \\
c_5 = -3(56\, nrot\, \pi - 15\dot{\gamma}_0\, T_f + (\mu^{or})^2\dot{\gamma}_0 T_f^3)/T_f^5; \\
c_6 = 2(70\, nrot\, \pi - 18\dot{\gamma}_0\, T_f + (\mu^{or})^2\dot{\gamma}_0 T_f^3)/T_f^6; \\
c_4 = -(80\, nrot\, \pi - 20\dot{\gamma}_0\, T_f + (\mu^{or})^2\dot{\gamma}_0 T_f^3)/(2T_f^7)\,.
\end{gathered} \tag{17}$$

Here $nrot$ is number of the complete revolutions of the tether during deployment;

Having differentiate twice the expression (16) with respect to time, one can obtain expressions for the first and second derivatives. Substituting the given parameters of the deployment system in the expressions (17), as a result of the solution of the initial value problem (14), one can obtain the time dependent law of the tether length change. This law provides deployment of the necessary length tether with alignment it along the local vertical. at the end.

Numerical example. Consider further implementation of the method of the program deployment of the space tether along the local vertical with use of its initial rotation.

The analysis of the initial value problem (14) shows that the angular velocity of the tether rotation $\dot{g}(t)$ cannot be negative for creation of the program law of change of the tether length, since the denominator in the equation (14) turns into zero and the solution of the problem has a singularity point at $\dot{g}(t) = -\mu^{or}$.

Accept the following values of the tether parameters for numerical example:

mass of the end bodies are identical and equal $m = 10$ kg;

initial length of the tether branch r_0 =2 m;

initial angular velocity $\dot{g}_0$=1 rad / s;

number of rotations before the end of the deployment $nrot = 70$;

duration of the deployment T_f = 3000 s.

Use the equations of Hill-Clohessy-Wiltshire (4) for simulation of the dynamics of the tether deployment in the centrifugal force field. There equations describe any spatial motion of the tether, and not just the motion in the orbital plane. The current length of the tether does not enter directly into these equations. The program for simulation of the dynamics of deployment is constructed as follows: after calculation of the coefficients of the power series (16), program values of the pitch angle $g(t)$ and its first two derivatives on time are calculated on each an integration step of the initial value problem (14). The obtained values are substituted in the right part of equation (14), and the current program value of the tether length is calculated. In parallel, the program value of the tension force is calculated on the basis of equation (3). This value is used in the HCW equations as the program control. It completely corresponds to program changes of the tether length.

The solving of the problem of creation of the program law of change of the tether length was made with the integration step of 0.01 s. The following results are obtained at this. The law of change of the tether length is shown in Fig. 3.

One can see here that the tether length increases slowly in the beginning, then the velocity of deployment significantly increases at the certain time interval and smoothly decreases to zero at the end. The law of program change of the velocity of deployment shows that it reaches the greatest level at the final stage of deployment, sharply decreasing to zero right at the end (see Fig. 4).

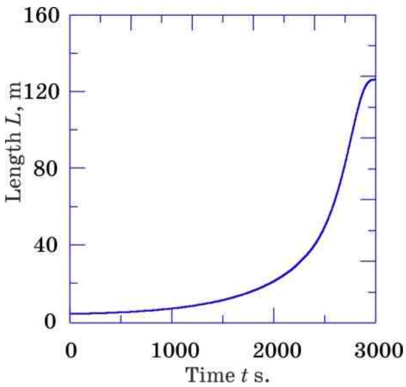

Fig. 3

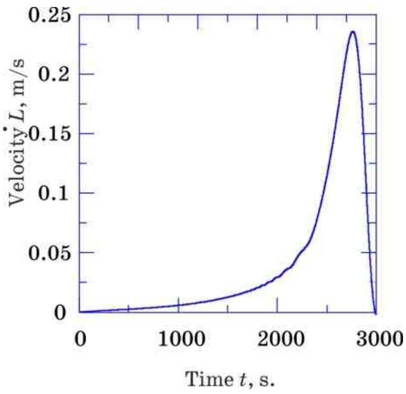

Fig. 4

The tether angle of rotation about the binormal to the orbit behaves as it is shown in Fig. 5.

Here it is possible to see the fast increase in the angle of rotation at the beginning of deployment and decrease in the angular velocity of rotation as the tether length increase.

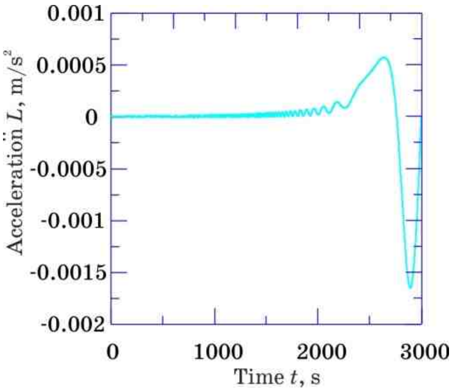

Fig. 5

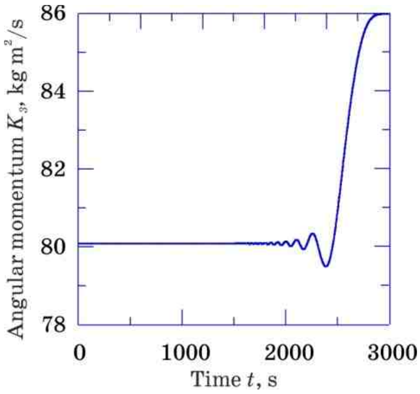

Fig. 6

It corresponds to the theorem of change of the angular momentum taking into account that the gravitational torque in the considered mode increases the total angular momentum of the tether. Nevertheless, the analysis of the value of the total angular momentum of the tether (Fig. 6) shows that at small sizes of the tether at the initial time the influence of the gravitational torque is a little noticeable.

Only at the very end of deployment this influence becomes more noticeable with each rotation. The gravitational torque makes the biggest contribution to the total angular momentum of the tether at the final time of deployment. The law of change of the gravitational torque vs. time is shown in Fig. 7.

Here it is visible that though amplitude of the gravitational torque begins to grow considerably from the instant t=1000 s, its total effect cannot be shown considerably since the tether length is changing slowly at the beginning and sign of the slowly changing gravitational torque is changing almost periodically. The maximum value of the gravitational torque arises on the finishing not rotational motion of the tether and turns in zero when the tether is aligned along the local vertical.

At deployment of the tether controlled by the found law, the program tension of the tether behaves as it is shown in Fig. 8.

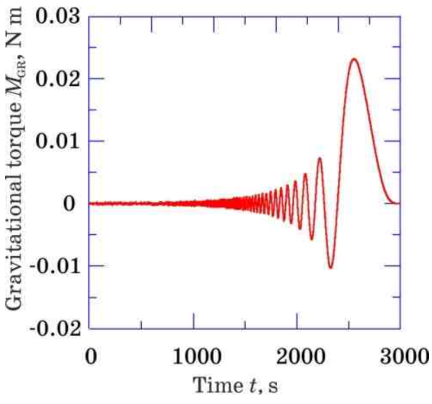

Fig. 7

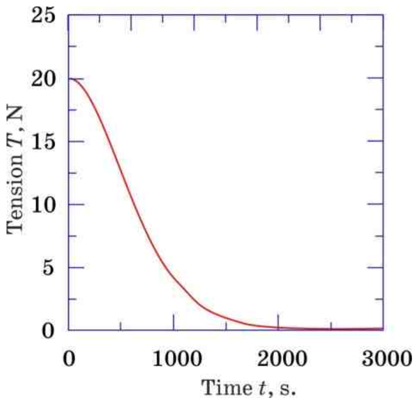

Fig. 8.

Naturally, the greatest force of tension in the tether at such a way of deployment arises at the initial instant, when the centrifugal forces acting on the end bodies are maximum. In the considered example, the maximum force does not exceed 20 N. Such a force without problems and noticeable deformations is perceived by thin threads made of modern materials. The minimum tension force in the tether corresponds to its state along the local vertical. While threads are fixed on the spacecraft, the tether tension is defined by the ratio of mass of the end body and spacecraft. According to [8], the tension force of the tether aligned along the local vertical is equal

$$T = 3m_R(\mu^{or})^2 L \ ,$$

where $m_R = m_1 m_2 / (m_1 + m_2)$, L is the tether length. In this case it is the tether branch length. As it is supposed that $m_1 << m_2$, $m_R = m_1$. If the deployed two-body tether separates from the spacecraft after joining of the threads in the point of their exit from the device of deployment, then $m_R = m_1 / 2$ and the tension tether remains the same, since the tether length in that case $L = 2r$. Figure 9 corresponds to the first case. The tether tension is equal to 0.0022 N at the end of deployment. Such a value is weakly visible in the graph.

Further simulation of the tether dynamics in the deployment mode was carried out according to the constructed law of change of the tether length in time after separation of the end body from the deployment device. The trajectory of motion of the end body of one branch tether is constructed on the plane $Cx_1^{or} y_1^{or}$ as a result of solving of the initial value problem for equations of the controlled motion of the tether in the form of Hill–Clohessy–Wiltshire (4) with the accepted initial conditions. This trajectory is shown in Fig. 9.

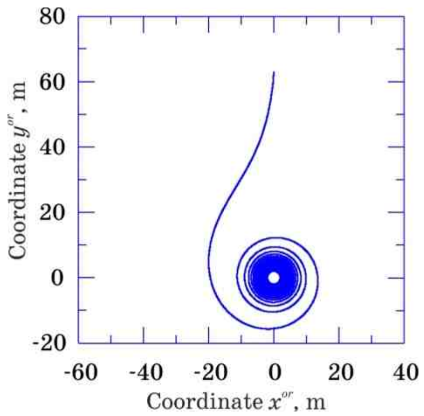

Fig. 9.

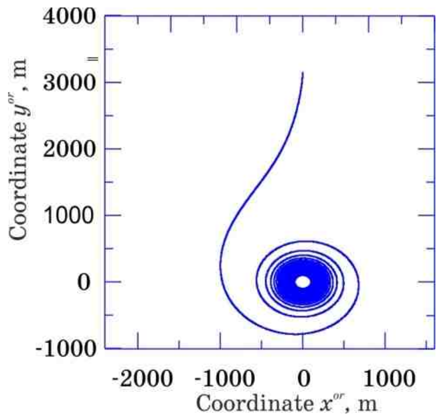

Fig. 10

It is visible in the Fig. 9 that at the initial period while length of the tether grows slowly and the gravitational torque is small, the step of the spiral motion of the end body is also small. On the last revolution, the end body significantly deviates towards negative values of the axis Cy_1^{or} and here the gravitational torque, maximum in size, completes deployment process, aligning the tether along the local vertical.

Naturally, the maximum tension of the tether arises at the initial stage of its rotation in the deployment device at such a mode of deployment. In the considered example this value is 20 N.

Numerous numerical experiments were made at various values of the system parameters. It was established that at such a way of deployment of the tether it is possible to reach significantly large lengths of the tether in principle. In Fig. 10, the trajectory of the end body of one branch is shown, which is implemented at deployment with the initial r_0 value equal 100 m and with the initial angular velocity of rotation 1 rad / s. Naturally, the deployment device of such sizes is unrealizable. On the other hand, it is possible to try to carry out preliminary deployment from the device having r_0 =2m to length of the branch of the rotating tether of 100 m. But at the same time, the angular velocity of the initial rotation must be about 25 rad/s. When masses of the end bodies are equal 10 kg, the tether tension is about 12500 N that creates serious problems. in spite of the fact that the tether thread can be made of variable cross section, having strengthened it on the sections of the thread, which are subjected to maximum tension.

Results of the error analysis of simulation of the tether deployment in the centrifugal force field are shown in Fig. 11. Here the blue line shows the value of the total angular momentum of the tether during deployment calculated both on changes of the phase variables and on the basis of the theorem about changes of the angular momentum under the influence of the gravitational torque. Both these lines in the figure coincide. Their values are defined by the left scale. The red line shows the difference in these values and characterizes errors of calculations.

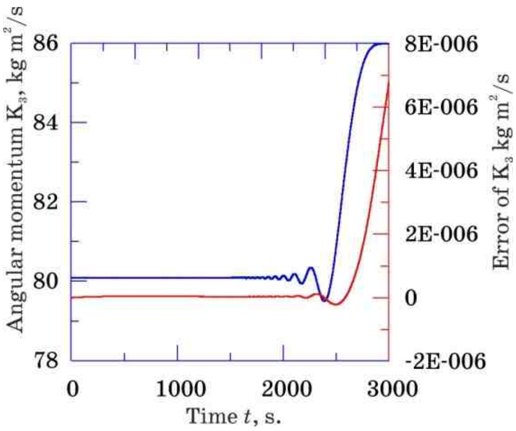

Fig. 11

They are defined by the right scale of the figure. The small values of the errors of the numerical simulation in comparison with the calculated values allows to consider the obtained results as reliable.

Conclusion. The research of the mode of deployment of the space tether in the centrifugal force field with alignment it at the end of deployment along the local vertical is conducted. The preliminary rotation of the tether about the binormal to the orbit is used for deployment. The development of the tether length control law is made at the beginning. This control law provides executing of the planned deployment. The numerical simulation of the dynamics of the tether deployment under control of the constructed program law is carried out at the second investigation phase. The novelty of the conducted research consists in creation of the program control law, which allows to deploy the tether along the local vertical for one stage without need to carry out further damping of libratory oscillations. The obtained results give the opportunity to determine the ranges of values of the deployment parameters allowing to carry out deployment of the considered type. The assessment of errors of the numerical simulation is carried out. The practical importance of the obtained results consists in the opportunity to carry out deployment of small tethers in the orbit with alignment them at the end of the mode along the local vertical for one stage with the tether length control.

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