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Mathematical approaches to particle size determination in closed grinding cycles are considered. The features of average particle size calculation for different fractions with account for the grinding kinetics are shown. Particle size calculation algorithms for the entire fraction range are proposed. Particular attention is paid to output determination for fractions of arbitrarily small particles. A particle size determination method based on a lognormal distribution function is shown. In choosing the mathematical approach, the process requirements are taken into account.

The basis of in-flow noncontact particle size control is the acoustic monitoring of the process and the established relationships between the particle size and the acoustic characteristics. The signal amplitude during material transportation in the energy carrier flow and jet grinding was found as a function of the particle size and grinding conditions. In order to determine the fractional composition of a mixture, the frequency characteristics of acoustic signals and their variation during the transportation of narrow fractions and mixtures were considered. The analysis of the amplitude-frequency characteristics of acoustic signals during the compressed-air transportation of narrow fractions in the jet mill channels confirmed the presence of signals with frequencies characteristic for each fraction. These frequencies were experimentally related to the particle size of a fraction in a mixture. These studies form a basis for a noncontact method of determining the particle size distribution of a material in an air flow, in particular in jet grinding. The results may be used for engineering and technological calculations in mineral dressing and the development of process equipment for the chemical industry, construction, mining, and metallurgy.

Keywords: *distribution law, size grade, acoustic signal, frequency, dispersion.*

(,) ,

$$q_1 = q_0(1 - \gamma_1), \quad (1)$$

$q_0 -$
 $\gamma -$

$$q_2 = q_0(1 - \gamma_1) + q_1(1 - \gamma_2) = q_0(1 - \gamma) + q_0(1 - \gamma_1)(1 - \gamma_2).$$

N

$$q_N = q_0 \sum_{i=1}^N \sum_{j=1}^i (1 - x_j)_i. \quad (2)$$

$$\gamma = \int_0^{d_{\max}} P(d) f(d) dx. \quad (3)$$

$$\gamma_1 = \gamma_2 = \gamma_3 = \dots = \gamma_n = \gamma. \quad (2)$$

$$(1 - x), \quad S = \frac{1 - (-x)^N}{x},$$

$$N \rightarrow \infty \quad S = \lim S = \lim \frac{1 - (-x)^N}{x} = 1/x.$$

$$q_N = q_0/\gamma. \quad (4)$$

[9].

$$d = d_0 \exp(-kt), \quad (5)$$

$k -$; $t -$; $d_0 -$

$t = t'$.

$$d_1 = d_0 \exp(-kt') = d_0 A, \quad A = e^{-kt'}$$

$$d_2 = d_1 A = d_0 A^2.$$

$$d_n = d_0 A^n.$$

$(1 - \gamma)$.

$$d_2 = \frac{d_0 A + d_0 A^2 (1 - \gamma)}{1 + (1 - \gamma)}$$

$$d_n = d_0 \frac{A + A^2 (1 - \gamma) + \dots + A^n (1 - \gamma)^{n-1}}{1 + (1 - \gamma) + \dots + (1 - \gamma)^{n-1}}. \quad (6)$$

(6)

$$\lim S_1 = \lim \frac{A(A^n (1 - \gamma)^n - 1)}{A(1 - \gamma) - 1} = \frac{A}{1 - A(1 - \gamma)},$$

$$\lim S_2 = \lim \frac{(1 - \gamma)^n - 1}{(1 - \gamma) - 1} = 1/\gamma.$$

$$d = d_0 \frac{A\gamma}{1 - A(1 - \gamma)}. \quad (7)$$

[9],

[9]

$$f(x) = F'(x) \quad 0 - \varepsilon, \quad \Phi(x) = F(x) \quad (0; 1)$$

$$\phi_H(Z), \quad \Phi_H(Z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad \Phi_H(Z),$$

$$\Phi_H(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^Z e^{-t^2/2} dt. \quad (8)$$

$x \rightarrow 0$

$$\phi(x) = \frac{1}{\sigma} \phi_H\left(\frac{1}{\sigma} \ln \frac{x}{\varepsilon}\right), \quad \Phi(x) = \Phi_H\left(\frac{1}{\sigma} \ln \frac{x}{\varepsilon}\right). \quad (9)$$

$$t = \frac{1}{\sigma} \ln \frac{x}{\varepsilon}.$$

$$f(x) \sim x \cdot \phi(x), \quad f(x) = C \frac{1}{\sqrt{2\pi\sigma}} e^{-t^2/2}.$$

$(a, b) \subset (0, \varepsilon),$

$$J_k(a, b) = \int_a^b x^{-k} f(x) dx.$$

$$0 - \varepsilon \quad (F(\varepsilon) = J_0(0, \varepsilon))$$

$$J_k(a, b) = F(\varepsilon) \frac{\exp\left\{\frac{\sigma^2}{2} [(k-1)^2 - 1]\right\}}{\varepsilon^k \Phi_H(-\sigma)} \left[\Phi_H\left(\frac{1}{\sigma} \ln \frac{b}{\varepsilon} + \sigma(k-1)\right) - \Phi_H\left(\frac{1}{\sigma} \ln \frac{a}{\varepsilon} + \sigma(k-1)\right) \right]. \quad (10)$$

$$F(x) = J_0(0, x),$$

$$F(x) = \frac{F(\varepsilon)}{\Phi_H(-\sigma)} \Phi_H\left(\frac{1}{\sigma} \ln \frac{x}{\varepsilon} - \sigma\right), \quad (11)$$

$f(x)$

$$f(x) = \frac{F(\varepsilon)e^{-\sigma^2/2}}{\varepsilon \cdot \sigma \cdot \Phi(-\sigma)\sqrt{2\pi}} e^{-t^2/2}. \quad (12)$$

$$(a, b) \quad Z(a, b) = \frac{J_0(a, b)}{J_1(a, b)},$$

$$Z(a, b) = \varepsilon \cdot e^{\sigma^2/2} \frac{\Phi_H\left(\frac{1}{\sigma} \ln \frac{b}{\varepsilon} - \sigma\right) - \Phi_H\left(\frac{1}{\sigma} \ln \frac{a}{\varepsilon} - \sigma\right)}{\Phi_H\left(\frac{1}{\sigma} \ln \frac{b}{\varepsilon}\right) - \Phi_H\left(\frac{1}{\sigma} \ln \frac{a}{\varepsilon}\right)}. \quad (13)$$

$(0, x)$

$$Z(0, x) = \varepsilon \cdot e^{\sigma^2/2} \frac{\Phi_H(t - \sigma) - \Phi_H(-\sigma)}{\Phi_H(t) - 0.5}. \quad (14)$$

(9)

$$F(\varepsilon) \quad 0 - \varepsilon,$$

$\sigma,$

v

$$v = 0,041,$$

$$v = 0,045.$$

$$(0 - 0,01)$$

p

$$p = \frac{F(0,01)}{F(\varepsilon)}, \quad p = \frac{\Phi_H\left(-\frac{1}{\dagger} \ln \frac{x}{2} - \dagger\right)}{\Phi_H(-\dagger)}.$$

$$\dagger = \dagger(p).$$

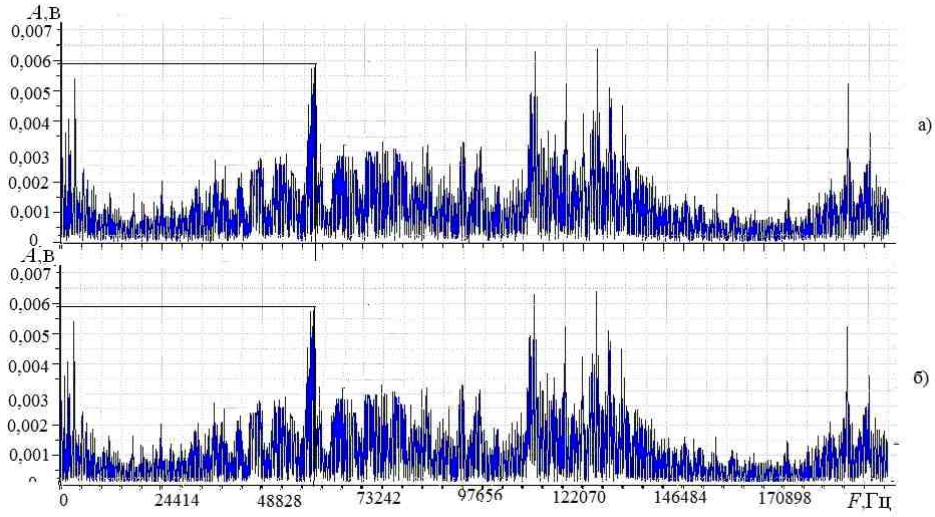
$$p \quad \dagger [10].$$

[8, 11].

[8].

[11]

(, ,). . 1 - 2

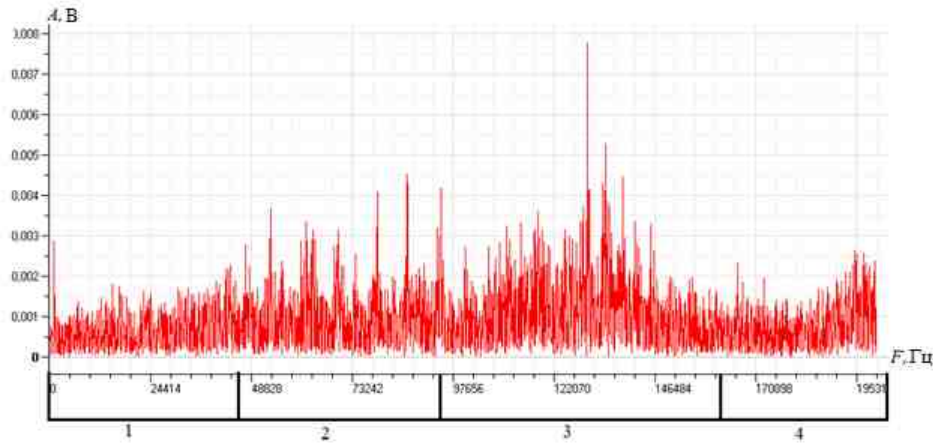


. 1 - -
 -0,63+0,4 () -0,4+0,315 (b)

(. . 2).

	1	2	3	4
, kHz	0 - 48	48 - 90	90 - 160	> 160

(48 - 90)



. 2 -

[8].

4

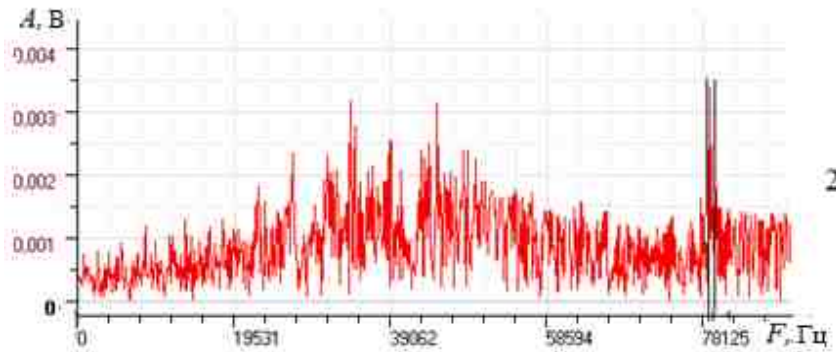
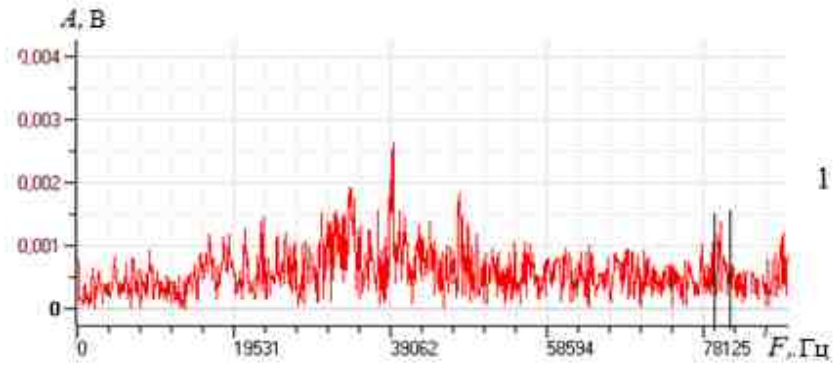
(, ,)

0 48 90 , -

[8].

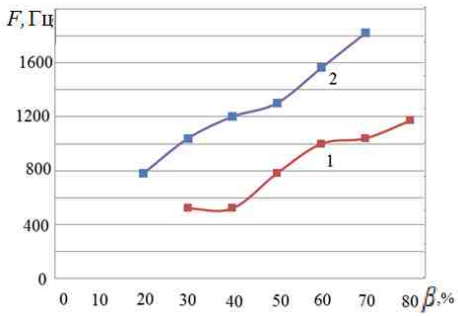
80 20 -

100 . .3 .4 -



1 - 70 -0,63 + 0,4 , 30 -1,0 + 0,63 ;
 2 - 30 -0,63 + 0,4 , 70 -1 + 0,63

.3 -



1 – $-0,315 + 0,2;$
 2 – $-0,4 + 0,315.$

. 4 –

2
30

70

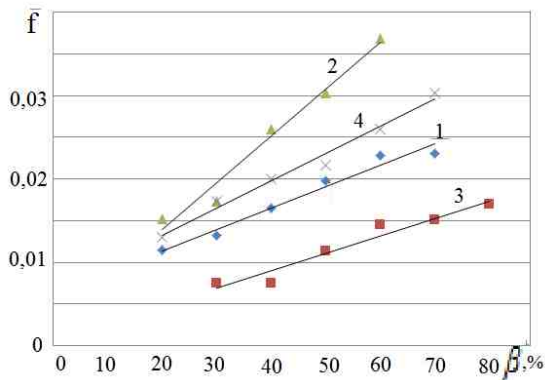
(79)

-0,63 + 0,4

(. . . 3).

. 5,

$$\bar{f} = aS + b, \quad a, b$$



: 1 – $-0,63 + 0,4;$

2 – $-0,4 + 0,315;$

3 – $-0,315 + 0,2;$

4 – $-0,4 + 0,315$

. 5 –

, $a -$

$$, \quad = \bar{f}' = \frac{df}{dS}.$$

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$$, \quad \bar{f} = aS + b \quad \bar{f}' = \frac{df}{dS} = k = d - p.$$

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