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MATHEMATICAL MODELING OF DETERMINATION THE IONOSPHERIC PLASMA CHARGED PARTICLES DENSITY BY ELECTRIC CURRENT MEASUREMENTS USING AN INSULATED PROBE SYSTEM

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Метою роботи є теоретичне обґрунтування можливості визначення концентрацій заряджених частинок іоносферної плазми за окремими вимірюваннями струмів ізольованої зондової системи в області насичення електронів. Склад іоносферної плазми моделюється іонами двох сортів з істотно різними масами і електронами, що забезпечують квазінейтральність плазми. Ізольована електрично від корпусу космічного апарата зондова система складається з циліндричних електродів – зонда й опорного електрода. Відношення площ збираючих струм поверхонь опорного електрода і зонда може бути істотно менше вимог теорії одиночного циліндричного зонду. Електроди поперечно обтікаються надзвуковим потоком беззіштовхувальної плазми. Поряд з основною плазмою з двохсортними іонами розглянута модельна плазма з модельними односортними іонами. Маса модельних іонів береться такою, щоб іонний струм насичення на циліндр збігся з іонним струмом насичення в потоці основної плазми.

На основі отриманого раніше асимптотичного розв'язку для струму насичення електронів у плазмі з односортними іонами знайдено розрахункові формули для визначення масового складу іонів і концентрації електронів за вимірюваннями зондових струмів. Погрішності отриманих формул в залежності від геометричних параметрів ізольованої зондової системи, потенціалів зсуву зонда щодо опорного електрода і точності вимірювань зондових струмів і потенціалів оцінено чисельно та аналітично. Показано, що при певному виборі налаштувань зондової системи і точності зондових вимірювань забезпечується достовірне визначення концентрацій заряджених частинок плазми з іонами двох сортів. Приведено апріорні оцінки впливу погрішностей вимірювань зондових струмів і потенціалів зсуву на достовірність визначення масового складу іонів і концентрації електронів іоносферної плазми.

Ключові слова: *двохсортні іони плазми, зондова система з циліндричними електродами, модельні односортні іони, математична модель збирання струму, достовірність визначення іонного складу і концентрації електронів.*

The goal of this work is to theoretically substantiate the possibility of determining the charged particle density in the ionospheric plasma by separately measuring the electric currents of an insulated probe system in the electron saturation region. The ionospheric plasma composition is modeled by two ion species with significantly different masses and electrons to keep the plasma quasi-neutrality. The probe system, which is electrically insulated from the spacecraft structure, consists of cylindrical electrodes: a probe and a reference electrode. The reference electrode to probe current-collecting area ratio can be significantly less than required by the single cylindrical probe theory. The electrodes are oriented transversely to a supersonic flow of a collisionless plasma. In addition to the main plasma with two ion species, a model plasma with a single ion species is considered. The mass of the model ions is such that the ion saturation current to the cylinder is the same for both plasma models.

Based on a previously obtained asymptotic solution for the electron saturation current in a plasma with a single ion species, computational formulas are found for determining the ion mass composition and the electron density by probe current measurements. The errors of the formulas are estimated numerically and analytically as a function of the probe system geometry, the bias potential of the probe relative to the reference electrode, and the accuracy of potential and current measurements. It is shown that a proper choice of the probe system settings and the accuracy of probe measurements assures a reliable determination of the charged particle densities in a plasma with two ion species. A priori estimates are presented for the effect of the current bias potential measurement errors on the reliability of determining the ion mass composition and the electron density of the ionospheric plasma.

Keywords*: two ion species plasma, probe system with cylindrical electrodes, model single-species ions, mathematical model of current collection, reliability of ion composition and electron density determination.*

Introduction. Diagnostics of low-temperature plasma using electric probes is widely used in studies of near-Earth and interplanetary space [1 – 4]. Among the types of electrical probes, the simplest and at the same time informative is a single stationary cylindrical Langmuir probe [5]. The spacecraft surface is usually used as a reference electrode for a single probe.

The development of a modern electronics has opened up new opportunities for research and monitoring the near-Earth space using micro- and nanosatellites.

However, the contacting plasma external conducting surface area of the nanosatellite is relatively small; this requires a significant reduction in the cylindrical probe size [4]. This circumstance and the strong rarefaction of the near-Earth plasma make it difficult to use the measuring scheme of a single Langmuir probe. In such a situation, we make use of measuring probe system isolated from the spacecraft body [1].

The Earth's ionosphere is a highly rarefied, weakly ionized gas mixture. The ion composition of the ionosphere varies significantly depending on the time of day (day/night), season (winter/summer), and levels of solar and geomagnetic activity [6]. At altitudes above 300 km, the main ionic components of the ionosphere can be divided into two groups of atomic ions: 1) oxygen O^+ and nitrogen N^+ ions of similar mass; 2) relatively light hydrogen H^+ and helium He^+ ions. During the day, the share of ions of the second group in the ionic composition does not exceed 10 %, but at night in winter for altitudes above 600 km it can exceed 50 %. In summer, the share of helium ions among relatively light ions does not exceed 10 %, and the share of nitrogen ions among heavier ions does not exceed ~20 %. Under such conditions, ionospheric plasma can be approximately considered as a weakly ionized gas mixture, the charged components of which consist of electrons and ions of two species with significantly different masses.

In [7, 8], a procedure is proposed for determining the parameters of low-temperature collisionless supersonic two ion species plasma flow by the current-voltage characteristics of isolated probe system with transversely oriented cylindrical electrodes. The procedure is based on the parametric identification of two current-voltage characteristics of the probe system, obtained at different electrodes areas ratios using a priori information about the experimental conditions. A mathematical model of current collection for an isolated probe system with an arbitrary ratio of electrode areas is developed.

When monitoring the state of the ionosphere in the vicinity of a spacecraft in orbit, it is important to reduce the required volume of transmitted information to determine the actual local plasma parameters. For example, in work [4], for this purpose, a reduction in the number of data points is used when transmitting current-voltage characteristics. In this regard, it makes sense to transmit information about current-voltage characteristics as individual measurements at such points in the transition and electron saturation regions, which are characteristic for mathematical models of collecting probe currents.

This article provides a theoretical substantiation for the possibility of determining the densities of charged particles of two ion species plasma from individual current measurements by the isolated probe system in the electron saturation region. Calculation relationships are developed and estimates are obtained considering the influence of the geometric parameters of the probe system and the errors in measuring electric current and voltage on the reliability of density determination.

Formulation of the problem. Modeling of probe measurements under ionospheric conditions is carried out using the weakly ionized gas mixture, the charged particles of which are electrons and singly charged atomic ions of oxygen O^+ and hydrogen H^+. The rarefaction of the gas is such that, for the probe measurement problem, the interaction of charged and neutral constituents can be neglected. The unperturbed plasma is quasi-neutral, and the velocity distribution of each charged particles species is Maxwellian.

The probe system consists of parallel cylindrical electrodes - a measuring electrode (probe) with a base radius r_p and a reference electrode with a base radius r_{cp}. The current-collecting surface of the reference electrode is variable [7]. The electrodes are placed transversely in the supersonic plasma flow with bulk velocity V. The base radii of electrodes are significantly smaller than their length, and the end surfaces are isolated from the plasma. We assume that electrostatic and gas-dynamic influence of the electrodes on each other in plasma is small, and there are no emission currents from the electrodes surfaces, the flow around electrodes is free-molecular and the influence of the magnetic field on the probe current is insignificant.

The main geometric parameter of the probe system is the ratio of their current-collecting surfaces (S_{cp} for reference electrode, S_p for probe): $S_s = S_{cp}/S_p$. And, the base radii of both electrodes satisfy the condition of applicability of the Langmuir asymptotic solution [9].

Taking into account the quasineutrality of the plasma, we characterize the ion composition by the parameter $\chi_n = n_{H^+}/(n_{H^+} + n_{O^+}) \equiv n_{H^+}/n_e$, where n_{H^+}, n_{O^+}, n_e are the densities of hydrogen ions, oxygen ions and electrons, respectively. Let's assume equal ion temperatures $T_{H^+} = T_{O^+} = T_i$.

The purpose is to develop a procedure for determining the densities of charged plasma particles from the results of electron saturation current measurements.

Mathematical model of current collection. A mathematical model of current collection by cylindrical electrodes of an isolated probe system in the two ion species plasma is developed in [7, 8]. The model is based on the Langmuir's classical asymptotic relations [10] for electric current on a long cylinder, analytical studies [11], and calculation results from [12 – 14] and assumption that the presence of ions of different species in a supersonic plasma flow does not change strongly the self-consistent electric field near the cylinder [9]. In dimensionless form, the total current (electron current is positive here and below) on the cylinder, which potential is φ relatively to undisturbed plasma, is estimated as follows:

$$\bar{I}_c(\varphi) = \bar{I}_e(\varphi) - 4\chi_n\sqrt{\mu_2/\beta}\cdot\bar{I}_{H^+}(\varphi) - (1-\chi_n)\sqrt{\mu_2/\beta}\cdot\bar{I}_{O^+}(\varphi), \quad S_i > 4, \qquad (1)$$

$$\bar{I}_e(\varphi) = \begin{cases} 2/\sqrt{\pi}\cdot\sqrt{\pi/4+\varphi}, & \varphi > 0; \\ \exp(\varphi), & \varphi \le 0 \end{cases},$$

$$\bar{I}_{H^+}(\varphi) = \begin{cases} \sqrt{2/\pi}\exp\left(-\beta\varphi + S_i^2/16\right), & \beta\varphi \ge S_i^2/16; \\ 2/\sqrt{\pi}\cdot\sqrt{1/2 + S_i^2/16 - \beta\varphi}, & \beta\varphi < S_i^2/16 \end{cases},$$

$$\bar{I}_{O^+}(\varphi) = \begin{cases} \sqrt{2/\pi}\exp\left(-\beta\varphi + S_i^2\right), & \beta\varphi \ge S_i^2; \\ 2/\sqrt{\pi}\sqrt{1/2 + S_i^2 - \beta\varphi}, & \beta\varphi < S_i^2 \end{cases},$$

where $\bar{I}_c$, $\bar{I}_e$ are the total and electron currents on a cylinder, respectively, normalized by the thermal electron current, $\bar{I}_{O^+}$, $\bar{I}_{H^+}$ are the currents of ions O^+, H^+ normalized by the thermal currents of ions of the corresponding species, $\varphi = eU/kT_e$ is the dimensionless electric potential (where U is a dimensional potential), k is the Boltzmann's constant, e is the unit charge , $\mu_2 = m_e / m_O$ is the mass ratio of charged particles, $\beta = T_e / T_i$ is the temperature ratio charged particles, $S_i = V / u_{O^+}$ is the velocity ratio for O^+ ions. The thermal current of particles of a type α is determined $I_{\sim,0} = j_{\sim} S_c$, where $j_\alpha = e n_\alpha u_\alpha / 2\sqrt{\pi}$ is the density of the thermal current, $u_\alpha = \sqrt{2kT_\alpha / m_\alpha}$ is the thermal velocity, T_α is the temperature, m_α is the mass of particles, S_c is the area of the current collecting surface of the cylinder. Here and below, the indices $\alpha = i$ refers the value to the ions, $\alpha = O^+$ – to atomic oxygen ions, $\alpha = H^+$ – to atomic hydrogen ions, $\alpha = e$ – to electrons.

In a supersonic plasma flow, the dimensionless current-voltage characteristic (CVC) of a probe system with transversely oriented cylindrical electrodes, at the bias potential φ_{iz} of the probe relative to the reference electrode is determined by

$$\bar{I}_p(\varphi_{iz}) = \bar{I}_c(\varphi_{iz} + \varphi_{cp}), \tag{2}$$

where φ_{cp} is the equilibrium potential of the reference electrode relative to the potential of the undisturbed plasma, corresponding to the bias potential φ_{iz}.

The equilibrium potential φ_{cp} is found from the current balance equation:

$$S_s \cdot \bar{I}_c(\varphi_{cp}) + \bar{I}_c(\varphi_{iz} + \varphi_{cp}) = 0 . \tag{3}$$

Thus, relations (1) – (3) determine the electrical and gas-dynamic interaction (CVC) in the "probe – plasma – reference electrode" system through the dimensionless parameters χ_n, μ_2, β, S_i, S_s and bias potential φ_{iz}. In the region of electron saturation at sufficiently high φ_{iz}, the current balance equation (3) has an analytical solution. However, for two ion species plasma, this solution is quite cumbersome.

Model mass of ions. Along with the considered above two ion species plasma, let's consider another model of plasma with one type of ions of mass m_{mod} and determine model plasma parameters using CVC of isolated probe system. Model plasma differs from two ion species plasma only in the ion composition, while other parameters (particles densities and temperatures, bulk velocity) and properties (quasineutrality, Maxwellian particles velocity distribution) remain.

In the case of a one component model plasma, the analytical solution of the current balance equation in the region of electron saturation leads to asymptotic relation [7]:

$$\bar{I}_p(\varphi_{iz}) \approx \frac{2}{\sqrt{\pi}} \cdot \sqrt{\frac{S_s^2 \mu_{mod}}{1 + S_s^2 \mu_{mod}}} \cdot \sqrt{\left(1/2 + S_{i\,mod}^2\right)/\beta + \pi/4 + \varphi_{iz}} , \tag{4}$$

where $\mu_{mod} = m_e / m_{mod}$ is the mass ratio of charged particles, $S_{i\,mod} = V / u_{i\,mod}$ is the ion velocity ratio, and $u_{i\,mod} = \sqrt{2kT_i / m_{mod}}$ is the thermal velocity of the model plasma ions.

On the basis of solution (4) let's develop for the electron saturation region an approximate current collection model that is close to the main model (1) – (3). For that, we select the model ions mass m_{mod} so that, at large negative potential φ of the cylinder relative to the undisturbed plasma, the ion current on the cylinder in the model plasma flow

$$I_{i\,mod}(\varphi) = j_{i\,mod} \cdot S_p \cdot \frac{2}{\sqrt{\pi}} \sqrt{1/2 + S_{i\,mod}^{\;2} - \beta\varphi} \qquad (5)$$

equals the ion current in a two ion species plasma $I_i^{(2)}(\varphi) = j_{H^+} \cdot S_p \cdot \bar{I}_{H^+}(\varphi) + j_{O^+} \cdot S_p \cdot \bar{I}_{O^+}(\varphi)$.

Equating the ion currents $I_{i\,mod}(\varphi)$ and $I_i^{(2)}(\varphi)$ at $\varphi << -\left(1/2 + S_i^{\;2}\right)/\beta$ we obtain the relation between the thermal ion currents densities in the model and main plasmas:

$$j_{i\,mod} \approx j_{H^+} + j_{O^+} .$$

From this, taking into account the accepted notation, we find

$$m_{mod} \approx \frac{m_O}{(3\chi_n + 1)^2}, \quad \mu_{mod} \approx (3\chi_n + 1)^2 \mu_2 . \qquad (6)$$

Fig. 1 represents the dependence of the relative error $\bar{\varepsilon}_{I_i} = \left(I_{i\,mod} - I_i^{(2)}\right)/I_i^{(2)}$ on the dimensionless cylinder potential φ, where $I_{i\,mod}$ is calculated by (5) for model ions of mass (6). The curves correspond to different values of the parameter χ_n = 0 (1), 0.1 (2), 0.3 (3), 0.5 (4), 0.7 (5). Within the framework of the current collection model (1) – (3), the error $\bar{\varepsilon}_{I_i}$ is the methodological error in calculating the ion current in a two ion species plasma using the one component plasma model with ion mass (6). The results show that the dependence of error of ion current calculation using one-component plasma model (4), (6) on χ_n is not monotonic; the largest error is achieved at $\chi_n \approx 0.3$ and it does not exceed 2% at $\varphi < -50$.

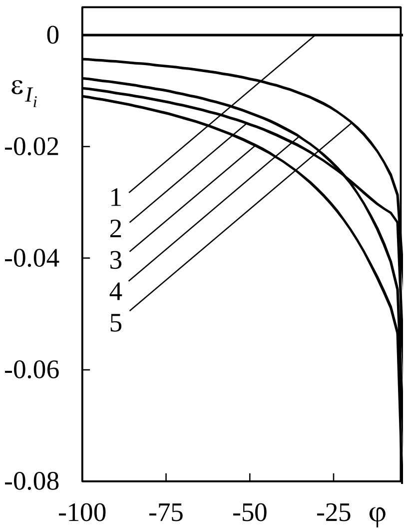

Fig. 1

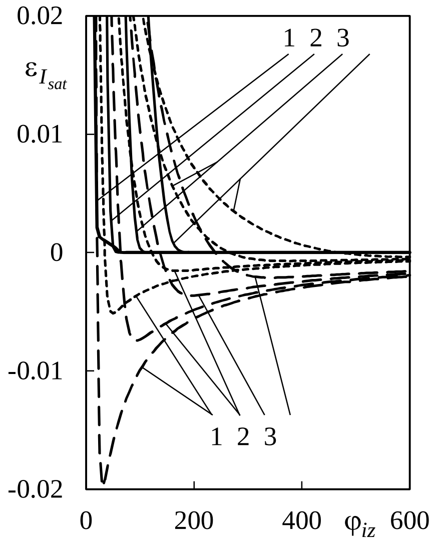

Fig. 2

Fig. 2 shows the dependence of relative difference $\bar{\varepsilon}_{I_{sat}} = \left(I_p^{(sat)} - I_p\right)/I_p$ between the probe electron saturation currents $I_p^{(sat)}$ and I_p, calculated by the approximate model (4), (6) and two ion species plasma model (1) – (3), respectively, on the bias potential φ_{iz}. Calculations are presented for the parameter χ_n = 0 (solid curves), 0.3 (dashed curves), 0.7 (dotted curves) for different values of the area ratio S_s =100 (1), 200 (2), 300 (3), 400 (4). Calculations are carried out at $S_i = 5$, $\beta = 1.3$, $T_e = 2800$ K, $n_e = 2 \cdot 10^{11}$ m^{-3}, that correspond to the flow conditions in the ionosphere at an altitude of about 700 km [6]. Within the current collection model (1) – (3), the error $\bar{\varepsilon}_{I_{sat}}$ is the methodological error of using model (4), (6) in the electron saturation region. The results show that an increase in the bias potential φ_{iz} leads to convergence of the one-component (4), (6) and two-component (1) – (3) models. At φ_{iz} >160, the error in calculating the probe current does not exceed 1 %.

Within the framework of the mathematical model of electron saturation current (4), the following relations for dimensional potentials and currents hold:

$$I'_p(U_{iz})/I_p(U_{iz}) \approx \frac{1}{2} \cdot \frac{1}{E_k + U_{iz}}, \tag{7}$$

$$\frac{I_p(U_{iz}) \cdot I'_p(U_{iz})}{I_p^2(U_{iz,2}) - I_p^2(U_{iz,1})} \approx \frac{1}{2} \cdot \frac{1}{U_{iz,2} - U_{iz,1}},$$

$$\frac{I_p^2(U_{iz,2}) + I_p^2(U_{iz,1})}{I_p^2(U_{iz,2}) - I_p^2(U_{iz,1})} \approx \frac{2E_k + (U_{iz,2} + U_{iz,1})}{U_{iz,2} - U_{iz,1}}, \tag{8}$$

where $E_k = \frac{m_{mod} \cdot V_i^2}{2e} + \frac{kT_i}{2e} + \frac{\pi k T_e}{4e}$.

Here, all the bias potentials U_{iz} belong to the range of applicability of the asymptotic solution (4):

$$U_{iz} > S_s^2 \mu_{mod} \cdot E_k - \left(S_s^2 \mu_{mod} + 1\right) \cdot \frac{\pi k T_e}{4e}. \tag{9}$$

Relations (7), (8) are to be used later to obtain estimates of errors in determining plasma parameters.

The inverse problem consists of determining plasma parameters (actually, parameters of the mathematical model of current collection (1) – (3)) by the results of measuring probe currents $I_p(U_{iz})$ with given geometric parameters of an isolated probe system. It is shown in [7] that, in contrast to a single Langmuir probe, the electron saturation current of an isolated probe system depends on the ion flow velocity S_i and the degree of plasma nonequilibrium β. We use this circumstance to determine the densities of charged plasma particles n_{H^+}, n_{O^+}, n_e using the one-component plasma model (4), (6) in the electron saturation region, where the measured current significantly exceeds the probe currents collected in the ion region of the CVC.

By definition, the densities of ions n_{H^+}, n_{O^+} and electrons n_e are related through χ_n by the relations:

$$n_{H^+} = \chi_n \cdot n_e, \; n_{O^+} = (1-\chi_n)\cdot n_e .$$

From relations (6) we find

$$\chi_n \approx \left(\sqrt{\mu_{mod}/\mu_2} - 1\right)\Big/ 3 .$$

If the parameter μ_{mod} is given with a relative error ε_μ, then for the parameter χ_n the relative error of calculation using this formula with an accuracy to the 3rd order terms is

$$\varepsilon_{\chi_n} \approx \frac{1}{2}\left(1+\frac{1}{3\cdot\overline{\chi}_n}\right)\cdot\left(\varepsilon_\mu - \frac{1}{4}\varepsilon_\mu{}^2 + \frac{1}{8}\varepsilon_\mu{}^3\right),$$

where $\overline{\chi}_n$ is the “exact” value of the parameter, $\overline{\chi}_n \neq 0$.

Thus, to determine the densities of charged plasma particles, it is necessary to estimate the parameter μ_{mod} and electron density n_e.

Determination of the parameter μ_{mod}**.** Writing down the asymptotic solution (4) for the bias potential U_{iz} for the areas ratios of $S_{s,1}$ and $S_{s,2}$, squaring the resulting two equations and resolving with respect to μ_{mod}, we have in dimensional form

$$\mu_{mod} \approx \frac{1}{S_{s,2}{}^2}\frac{I_{p,1}{}^2\cdot p_s{}^2 - I_{p,2}{}^2}{I_{p,2}{}^2 - I_{p,1}{}^2}, \; p_s = S_{s,2}\big/S_{s,1} > 1. \quad (10)$$

Here, $I_{p,1}$, $I_{p,2}$ are probe currents corresponding, within the framework of the mathematical model (4), (6), to the bias potential U_{iz} from the electron saturation region (9) at electrodes areas ratios $S_s = S_{s,1}$ and $S_s = S_{s,2}$, respectively. Note that formula (10) determines the parameter μ_{mod} only through the dimensional currents and does not explicitly depend on other parameters of the plasma flow.

Fig. 3 represents the dependence of the relative error $\overline{\varepsilon}_\mu = (\mu_{mod} - \overline{\mu}_{mod})/\overline{\mu}_{mod}$ on the bias potential U_{iz} (volts), where μ_{mod} is calculated using (10) at the exact values of probe currents $I_{p,1}$, $I_{p,2}$, corresponding to the mathematical model (1) – (3), $\overline{\mu}_{mod}$ is the mass of model ions calculated by (6). The results are shown for various χ_n at the area ratio $S_{s,1}$=100 and p_s=2 (solid curves), 4 (dashed curves). The curves correspond to: χ_n=0.1 (1), 0.3 (2), 0.5 (3), 0.7 (4). Within the framework of the current collection model (1) – (3), the error $\overline{\varepsilon}_\mu$ is a methodological error of calculating the parameter μ_{mod} using formula (10). The results obtained show that an increase in the bias potential U_{iz} and geometric parameter p_s leads to a monotonic decrease in $\overline{\varepsilon}_\mu$. At bias potentials $U_{iz} < 50$ V, the methodological error $\bar{\varepsilon}_\mu$ increases sharply, which makes it difficult to adequately determine the effective ions mass using model (4), (6). For a probe system with $S_{s,1}$=100 and p_s=4 at bias potentials more than ~50 V, the methodological error $\bar{\varepsilon}_\mu$ does not exceed 10 %, and at U_{iz} greater than ~100 V the $\bar{\varepsilon}_\mu$ does not exceed 1.5 %.

Determination of the electron density. From the asymptotic solution (4) in dimensional terms for bias potentials U_{iz} and $U_{iz} + dU$ from the electron saturation region (9) at the electrodes areas ratio S_s, squaring the resulting two equations and resolving with respect to n_e, we find

$$n_e \approx \frac{\pi}{eS_\text{p}}\sqrt{\frac{m_e}{2e}}\sqrt{1+\frac{1}{S_s{}^2 \cdot \mu_{mod}}} \cdot \sqrt{\frac{\left[I_p\left(U_{iz}+dU\right)\right]^2-\left[I_p\left(U_{iz}\right)\right]^2}{dU}}\,. \qquad (11)$$

Here $I_p\left(U_{iz}\right)$ is the probe current, corresponding within the framework of the mathematical model (4), (6) to the bias potential U_{iz}; $dU > 0$ is the increment of the bias potential.

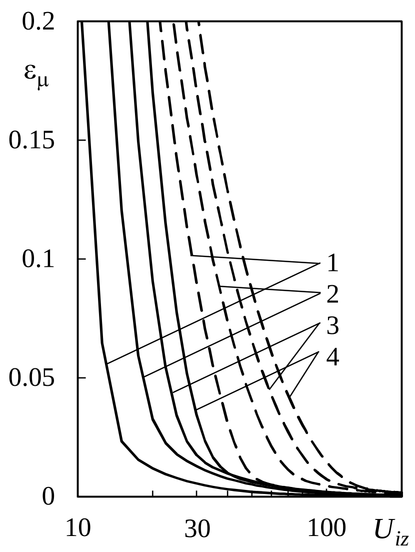

Fig. 3

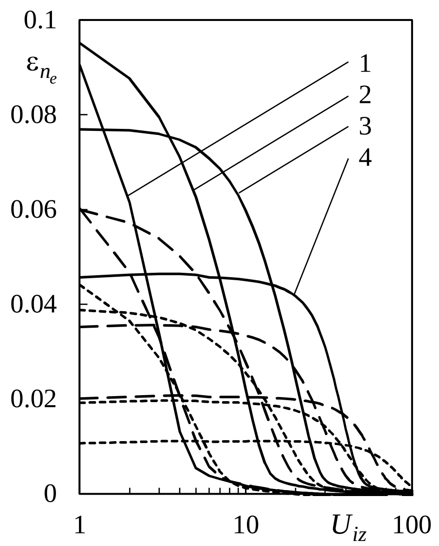

Fig. 4

Fig. 4 shows the dependence of the relative error $\bar{\varepsilon}_n = \left(n_e - \bar{n}_e\right)/\bar{n}_e$ on the bias potential U_{iz} (in volts), where n_e is the calculated by (11) and (10) using probe currents I_p, $I_{p,1}$, $I_{p,2}$, that correspond to the mathematical model (1) – (3), and $\bar{n}_e$ is the exact value of the parameter. The results are obtained for χ_n = 0.1 (solid curves), 0.3 (dashed curves), 0.5 (dotted curves) at different values of the area ratio S_s = 100 (1), 200 (2), 300 (3), 400 (4). Calculations are performed with dU = 20 V, $S_{s,1}$ = 100 and p_s = 4, bias potential when calculating μ_{mod} using (10) is 100 V. In the framework of the current collection model (1) – (3), the error $\bar{\varepsilon}_n$ is a methodological error of calculating the electron density n_e by (11) and (10). The calculation results show that an increase in the bias potential U_{iz} leads to a monotonic decrease in the error $\bar{\varepsilon}_n$ for all electrodes areas ratios $S_s \geq 100$. The greatest error $\bar{\varepsilon}_n$ for all S_s is achieved at $U_{iz} < 50$ V and does not exceed 10 %. In this case, the largest value of $\bar{\varepsilon}_n$ decreases as S_s increases, and at $S_s = 400$ the methodological error of formula (11) is $\bar{\varepsilon}_n \approx 1\,\%$.

Probe measurements errors. We consider the influence of errors in measuring currents and voltages by an isolated probe system on the reliability of determining the densities of charged particles within the framework of model (4), (6). Let the bias potentials U_{iz} and the corresponding probe currents $I_p(U_{iz})$ be given as approximate values

$$\tilde{U}_{iz} = U_{iz}\left(1 + \tilde{\varepsilon}_U\right),\ \tilde{I}_p\left(\tilde{U}_{iz}\right) = I_p\left(\tilde{U}_{iz}\right)\left(1 + \tilde{\varepsilon}_I\right),$$

where $\tilde{\varepsilon}_U$, $\tilde{\varepsilon}_I$ are random variables on the intervals $[-\varepsilon_U, \varepsilon_U]$, $[-\varepsilon_I, \varepsilon_I]$, respectively; $\varepsilon_U > 0$, $\varepsilon_I > 0$ are the maximum relative errors of the corresponding quantities. Here and below, the tilde sign indicates the approximate values of the corresponding quantities. Within the framework of the considered mathematical model (4), the electron saturation current $I_p(U_{iz})$ is approximated by a value $\tilde{I}_p\left(\tilde{U}_{iz}\right)$ with a relative error $\tilde{\varepsilon}_{I,U} = \tilde{\varepsilon}_I + I'_p(U_{iz}) / I_p(U_{iz}) \cdot U_{iz} \cdot \tilde{\varepsilon}_U$. Taking into account relation (7) and the accepted notation, we can write

$$\tilde{I}_p\left(\tilde{U}_{iz}\right) = I_p(U_{iz}) \cdot \left(1 + \tilde{\varepsilon}_{I,U}\right),\ \text{where}\ \tilde{\varepsilon}_{I,U} = \tilde{\varepsilon}_I + \frac{1}{2}\frac{U_{iz}}{E_K + U_{iz}}\tilde{\varepsilon}_U\,.$$

Error in determining the parameter μ_{mod}. Let us consider the influence of probe measurement errors and geometric parameters p_s, $S_{s,1}$ on the reliability of determining μ_{mod} using (10). Substituting approximate values into the calculation formula (10), after some straightforward transformations we have:

$$\varepsilon_\mu = \sup_{\tilde{\varepsilon}_{I,U}}\left|\frac{\mu_{mod} - \bar{\mu}_{mod}}{\bar{\mu}_{mod}}\right| \approx 4\frac{\dfrac{\left(p_s^2 \cdot S_{s,1}^2\bar{\mu}_{mod} + 1\right)}{\left(p_s^2 - 1\right)}\dfrac{S_{s,1}^2\bar{\mu}_{mod} + 1}{S_{s,1}^2\bar{\mu}_{mod}}\varepsilon_{I,U}}{1 - 2\dfrac{p_s^2}{p_s^2 - 1}\left(2S_{s,1}^2\bar{\mu}_{mod} + \dfrac{\left(p_s^2 + 1\right)}{p_s^2}\right)\varepsilon_{I,U} + \varepsilon_{I,U}^2}, \quad (13)$$

where ε_μ is the maximum relative error in determining the considered parameter, μ_{mod} is the result of calculations by (10) at probe currents $\tilde{I}_{p,1}\left(\tilde{U}_{iz}\right)$ and $\tilde{I}_{p,2}\left(\tilde{U}_{iz}\right)$, $\bar{\mu}_{mod}$ is the value of the parameter μ_{mod} calculated by (6), $\pounds_{I,U} = \pounds_I +$ $U_{iz} / \left(E_K + U_{iz}\right) \cdot \pounds_U / 2$ is the maximum relative error in measuring electron saturation currents $I_{p,1}(U_{iz})$, $I_{p,2}(U_{iz})$.

Relationship (13) characterizes the influence of geometric parameters $S_{s,1}$, p_s of the probe system and the maximum relative error $\varepsilon_{I,U}$ of measuring electron saturation currents on the reliability of determining the parameter μ_{mod} from

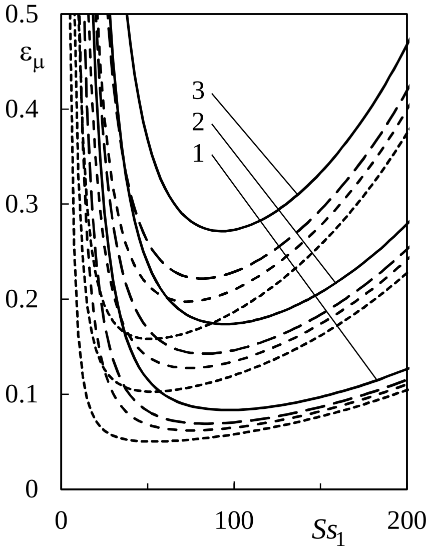

Fig. 5

(10). Fig. 5 represents the results of calculation ε_μ depending on the electrodes areas ratio $S_{s,1}$ at $\varepsilon_{I,U} = 0.01$ (curves 1), 0.02 (curves 2), 0.03 (curves 3). For each $\varepsilon_{I,U}$ curves of different types correspond to $p_s = 3$ (solid), 4 (long dashed), 5 (short dashed), 10 (dotted).

The calculation results show that a decrease in $S_{s,1}$ from ~50 results to a sharp increase in ε_μ for all values of p_s. As p_s increases, the error ε_μ decreases monotonically for all $\varepsilon_{I,U}$. At $S_{s,1} \geq 100$ and $p_s \geq 5$, an increase in p_s doesn't cause a significant decrease in ε_μ.

Taking into account the discussed above influence of $S_{s,1}$ and U_{iz} on the methodological error £«, we conclude that for estimating the ions composition it is reasonable to take p_s in the range from 3 to 5, $S_{s,1}$ from 60 to 100, $U_{iz} > 40$ V. In this case, the maximum relative error in measuring the probe current and bias potential should not exceed ~ 1 %.

Expanding the right-hand side of (13) into a series in $\varepsilon_{I,U}$ and keeping the 1st order terms, we obtain an estimate for the maximum relative error of the calculation formula (10)

$$\left|\frac{\mu_{mod} - \bar{\mu}_{mod}}{\bar{\mu}_{mod}}\right| \leq \varepsilon_\mu \approx 4\frac{\left(p_s{}^2 \cdot S_{s,1}{}^2\bar{\mu}_{mod} + 1\right)}{\left(p_s{}^2 - 1\right)}\frac{S_{s,1}{}^2\bar{\mu}_{mod} + 1}{S_{s,1}{}^2\bar{\mu}_{mod}}\left(\varepsilon_I + \frac{1}{2}\frac{U_{iz}}{E_K + U_{iz}}\varepsilon_U\right). \quad (14)$$

The above estimate and the data presented in Fig. 3 allow one to select the geometric parameters of the probe system, the bias potential and the necessary measurement accuracy for adequate estimation of the ion composition.

Error in determining the electron density. Let's study the influence of probe measurement errors and the geometric parameter S_s on the error in determining the parameter n_e using formula (11) within the framework of model (4), (6). We assume that μ_{mod} in the model (4), (6) is known with a relative error ε_μ, so that $\tilde{\mu}_{mod} = \bar{\mu}_{mod}\left(1 + \tilde{\varepsilon}_\mu\right)$, $\tilde{\varepsilon}_\mu \in \left[-\varepsilon_\mu, \varepsilon_\mu\right]$, $\bar{\mu}_{mod}$ is the value of μ_{mod} calculated by (6).

Substituting approximate values of $\tilde{\mu}_{mod}$, $\tilde{U}_{iz}$, $d\tilde{U}$, $\tilde{I}_p\left(\tilde{U}_{iz}\right)$ into (11), after straightforward transformations using (8) and the accepted notation, neglecting the second order members, we obtain:

$$\left|\frac{\tilde{n}_e - n_e}{n_e}\right| \leq \varepsilon_{n_e} \approx \frac{2E_k + \left(2U_{iz} + dU\right)}{dU} \cdot \varepsilon_I + \frac{1}{2\left(S_s{}^2\bar{\mu}_{mod} + 1\right)} \cdot \varepsilon_\mu, \quad (15)$$

where ε_{n_e} is the maximum relative error in determining the parameter n_e using formula (11), and the bias potential U_{iz} belongs to region (9). As we see, with an increase in the electrodes areas ratio S_s, the influence of uncertainty in μ_{mod} decreases. Therefore, when determining the electron density using formula (12), we take the largest value of the electrodes areas ratio $S_s = S_{s,2}$ used in determining the parameter μ_{mod}.

Substituting the estimated ε_μ from (14) into (15), taking into account (6), we obtain

$$\begin{aligned}\varepsilon_{n_e} \approx &\left[\frac{2E_k + (2U_{iz} + dU)}{dU} + \frac{2}{(p_s{}^2 - 1)}\left(1 + \frac{1}{(3\chi_n + 1)^2 S_{s,1}{}^2 \mu_2}\right)\right] \cdot \varepsilon_I + \\ &+ \frac{1}{(p_s{}^2 - 1)} \cdot \left(1 + \frac{1}{(3\chi_n + 1)^2 S_{s,1}{}^2 \mu_2}\right) \cdot \frac{U_{iz}{}^*}{E_K + U_{iz}{}^*} \cdot \varepsilon_U\end{aligned} . \qquad (16)$$

Here, $U_{iz}{}^*$ is the probe bias potential used to determine the parameter μ_{mod}.

A priori estimates of the reliability of determining the parameter μ_{mod} and electron density under the ionospheric conditions can be obtained using the data from the model of Ionosphere IRI-2020 [6]. According to that model, the mass fraction of atomic hydrogen ions χ_n can change from 0 to ~0.9 depending on the season and time of day at altitudes from 300 km to 900 km. In this case, the parameter $E_k = m_{mod} V_i{}^2 / 2e + kT_i / 2e + \pi k T_e / 4e$ in the model (4), (6) varies from ~0.5 eV to ~5 eV. Charged particle mass ratio $\mu_2 = m_e / m_O \approx 3.43 \cdot 10^{-5}$. Then, at $S_{s,1} \approx 100$, $p_s \geq 4$ and $dU > 20$, taking into account the obvious $1/(3\chi_n + 1)^2 \leq 1$, from (14) and (16) we obtain

$$\varepsilon_\mu < \left[5.5 + 1.5 \cdot (3\chi_n + 1)^2\right] \cdot \left(\varepsilon_I + \frac{1}{2}\varepsilon_U\right),$$

$$\varepsilon_{n_e} < 2\frac{U_{iz} + dU}{dU}\varepsilon_I + \frac{4}{15}\varepsilon_U .$$

From condition (9) follows the constraint for bias potential:

$$U_{iz} > 1.64 \cdot (3\chi_n + 1)^2 \cdot (S_s / 100)^2 .$$

The presented estimates of the maximum relative errors, together with the results in Fig. 3, 4 allow one to select the proper geometric parameters of the probe system, the values of the bias potential U_{iz}, potential increment dU and evaluate the necessary measurement accuracy for adequate determination of the parameter μ_{mod} and electron density under ionospheric conditions.

Conclusions. The possibility of determining the densities of two ion species plasma particles from individual electric current measurements in the electron saturation region using an isolated probe system is theoretically substantiated. The procedure for determining densities is implemented using a model of one-component plasma. Based on the asymptotic solution for the electron saturation current in the one-component plasma, an approximate model of the electron saturation current in the two ion species plasma is developed. It is shown that as the bias potential of the probe relative to the reference electrode increases, the approximate model of current collection in the one-component plasma approaches that in the two ion species plasma for any ion composition.

Calculation formulas are found to determine the ions mass composition and electron density from probe currents measurements in a supersonic two ion species plasma flow. Numerical estimates of the methodological errors of the calculation formulas are obtained depending on the electrodes areas ratio and the bias potential of the probe relative to the reference electrode. The errors in the calculation formulas for determining charged particles densities depending on the parameters of the probe system and the accuracy of the probe measurements are estimated analytically. The probe system parameters and the required accuracy of probe measurements are specified for reliable determination of the two ion species plasma particles densities. A priori estimates of the influence of the current and bias potential measurement errors on the reliability of determining the ions mass composition and the electron density in the ionospheric plasma are obtained.

The results obtained can be used in the diagnostics of ionospheric plasma.

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