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DETERMINATION OF PLASMA PARAMETERS IN A JET OF A GAS-DISCHARGE SOURCE USING AN ISOLATED PROBE SYSTEM WITH CYLINDRICAL ELECTRODES

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Метою роботи є розробка процедури визначення ступеня дисоціації іонів і концентрації електронів у надзвуковому струмені газорозрядного джерела беззіштовхувальної плазми за результатами вимірювань струму, що збирається ізольованою зондовою системою з поперечно-обтічними циліндричними електродами. На основі отриманих раніше математичної моделі збирання струму ізольованою зондовою системою й асимптотичного розв'язку для зондового струму в області насичення електронів отримані нові розрахункові формули для визначення параметрів плазми. Показано, що в порівнянні з одиночним зондом Ленгмюра ізольована зондова система має більш високу інформативність при діагностиці струменя газорозрядного джерела лабораторної плазми.

Проведено числові дослідження впливу відносини площин зонда і опорного електрода, що збирають струм, і погрішностей зондових вимірювань на визначення параметрів плазми. У рамках прийнятої математичної моделі збирання струму виконаний аналіз впливу геометричних параметрів ізольованої зондової системи на методичну погрішність при використанні для відновлення параметрів плазми асимптотичного розв'язку для зондового струму в області насичення електронів. Для процедури відновлення ступеня дисоціації іонів знайдені оптимальні для практичного використання потенціали зсуву і геометричні параметри ізольованої зондової системи – відносини площин зонда й опорного електрода. У рамках прийнятих припущень отримані оцінки вірогідності відновлення ступеня дисоціації іонів і концентрації електронів у залежності від геометричних параметрів ізольованої зондової системи, точності вимірювання зондових струмів і потенціалів зсуву зонда щодо потенціалу опорного електрода.

Отримані результати можуть бути використані в діагностиці лабораторної плазми газорозрядного джерела з прискоренням іонів в електричному полі струменя.

Ключові слова: *струмінь беззіштовхувальної плазми, ступінь дисоціації іонів, концентрація електронів, математична модель збирання струму, оцінки погрішності відновлення параметрів.*

The aim of this work is to develop a procedure for determining the ion dissociation degree and the electron density in a supersonic jet of a gas-discharge source of collisionless plasma from the results of measurements of the current collected by an insulated probe system with transversely oriented cylindrical electrodes. Based on a mathematical model of current collection by an insulated probe system and an asymptotic solution for the probe current in the electron saturation region obtained previously, new computational formulas for plasma parameter determination are derived. It is shown that, in comparison with a single Langmuir probe, an insulated probe system provides more information in diagnosing a jet of a gas-discharge source of laboratory plasma.

The effect of the probe to reference electrode current collection area ratio and the probe measurement errors on the plasma parameter determination accuracy is studied numerically. Within the framework of the mathematical model of current collection, an analysis is made of the effect of the geometrical parameters of the insulated probe system on the method error in plasma parameter determination using the asymptotic solution for the probe current in the electron saturation region. For the determination of the ion dissociation degree, optimal values of the insulated probe system's bias potentials and geometrical parameters (probe to reference electrode area ratio) are found. For the adopted assumptions, the reliability of ion dissociation degree and electron density determination is estimated as a function of the geometrical parameters of the insulated probe system and the probe current and probe potential (relative to the reference electrode) measurement accuracy.

The obtained results may be used in the diagnostics of the laboratory plasma of a gas-discharge source with ion acceleration in the electric field of the jet.

Keywords: *collisionless plasma jet, ion dissociation degree, electron density, mathematical model of current collection, estimation of parameter determination error.*

Introduction. Currently, one of the main methods for diagnosing low-temperature rarefied plasma is the method of a stationary electric probe. The simplicity of the design and the possibility of determining the main parameters of the plasma make the cylindrical Langmuir probe a reliable tool for laboratory testing of structural elements and on-board equipment of spacecraft.

In physical modeling of ionospheric conditions, gas-discharge sources (GDS) of plasma with ion acceleration in the electric field of the jet are used [1]. Adequate laboratory modeling of the interaction of spacecraft structural elements with ionospheric plasma suggests a complete diagnosis of the supersonic dissociated GDS jet. Therefore, increasing the information content and accuracy of laboratory plasma diagnostics is an important element of physical modeling.

The article [2] considers an approach to diagnosing a GDS jet using an isolated probe system (IPS) with cylindrical electrodes oriented transversely to the stream direction. Under the condition that the squares of the mass velocity of atomic and molecular ions in the jet are inversely proportional to their masses, an asymptotic solution for the electron saturation current is obtained. A relation between the degree of ion dissociation and the collected probe currents at different ratios of the areas of the probe and the reference electrode is found. A procedure of determining the degree of ion dissociation is proposed on the basis of probe measurements in the jet core region. It is shown that the problems of determining the degree of ion dissociation and determining other plasma parameters, such as temperature and density of electrons, temperature and velocity if ions) are separated.

This article is a continuation of [2]. Under the accepted assumptions, new relationships are obtained between the plasma parameters and probe currents measured by cylindrical electrodes on spacecraft. The influence of the geometric parameters of the probe system and currents and voltages measurement errors on the reliability of determining the electron density and the degree of ion dissociation is studied. The optimal for practical use geometrical parameter of the IPS, i. e. the ratio of the areas of the probe and the reference electrode, is determined.

Formulation of the problem. Let's consider a model of a probe measuring system with cylindrical electrodes transversely placed in a supersonic plasma flow of a dissociated diatomic gas. The plasma flow is formed by the GDS with the acceleration of ions in the electric field of the jet flowing into the vacuum chamber [1, 3].

Measuring (probe) and reference electrodes are electrically isolated from the body of the vacuum chamber. The reference electrode consists of the series of identical cylinders located in parallel, each of which can be connected or disconnected from the measurement electrical circuit. It's assumed that the probe base radius r_p and the reference electrode base radius r_{cp} are significantly less than electrodes length, the ends of the electrodes are isolated from the plasma, the electrostatic and gas-dynamic influence of the electrodes on each other in plasma is small, and there are no emission currents from the electrode surfaces.

The GDS jet contains neutrals, positive singly charged ions of two types (molecular ions of mass m_i and atomic ions of mass $m_i/2$) and electrons. It's assumed that the flow around the electrodes is collisionless, the influence of the magnetic field on the probe current isn't significant, and velocity distribution of particles of each type in the unperturbed plasma is Maxwellian. Since the ions are accelerated in the electric field of the jet, the mass velocities of atomic $V_{i,1}$ and molecular $V_{i,2}$ ions satisfy the relation $V_{i,1}/V_{i,2} = \sqrt{2}$.

It follows from the condition of plasma quasi-neutrality that $n_{i,1} + n_{i,2} = n_e$, where $n_{i,1}$ and $n_{i,2}$ are the density of atomic and molecular ions, respectively, and n_e is the density of electrons. The degree of dissociation of ions in the jet is characterized by the parameter $\eta = n_{i,1}/n_e$. The temperatures of atomic and molecular ions are assumed to be equal $T_{i,1} = T_{i,2} = T_i$.

It is required to develop a procedure for determining the parameters η and n_e in the core region of the jet by the results of measurement of the probe current in the region of electron saturation, i. e. to obtain calculation formulas and estimates of the error in determining the parameters depending on the geometrical parameters of the IPS and the values of the bias potential of the probe relative to the reference electrode.

Mathematical model of current collection. In a supersonic flow of dissociated rarefied GDS plasma, the probe current of the IPS in dimensionless quantities at the probe bias potential φ_{iz} relative to the reference electrode is determined by the formula [2]

$$\bar{I}_p(\varphi_{iz}) = \bar{I}_e(\varphi_{iz} + \varphi_{cp}) + \bar{I}_i(\varphi_{iz} + \varphi_{cp}), \qquad (1)$$

where the equilibrium potential $\varphi_{cp} = \varphi_{cp}(\varphi_{iz})$ of the reference electrode relative to the potential of the unperturbed plasma is found from the current balance equation:

$$S_s \cdot \left[\bar{I}_e(\varphi_{cp}) + \bar{I}_i(\varphi_{cp})\right] + \left[\bar{I}_e(\varphi_{iz} + \varphi_{cp}) + \bar{I}_i(\varphi_{iz} + \varphi_{cp})\right] = 0. \qquad (2)$$

Here, S_s is the geometrical parameter of the probe system: $S_s = S_{cp}/S_p$, S_{cp} is the area of the current-collecting surface of the reference electrode, and S_p is the surface area of the probe ($S_p << S_{cp}$).

The functional dependences of the electron $\bar{I}_e(\varphi)$ and ion $\bar{I}_i(\varphi)$ currents to the cylindrical electrode are given by the relations [2]

$$\bar{I}_e(\varphi) = \begin{cases} 2/\sqrt{\pi} \cdot \sqrt{\pi/4 + \varphi}, & \varphi > 0; \\ \exp(\varphi), & \varphi \le 0 \end{cases},$$

$$\bar{I}_i(\varphi) = -(1 + 0.414\eta)\sqrt{\frac{\mu}{\beta}} \begin{cases} \sqrt{2/\pi}\exp\left(-\beta\varphi + S_i^{\,2}\right), & \varphi \ge S_i^{\,2}/\beta; \\ 2/\sqrt{\pi}\sqrt{1/2 + S_i^{\,2} - \beta\varphi}, & \varphi < S_i^{\,2}/\beta \end{cases},$$

where $\varphi = eU/kT_e$ is the dimensionless electric potential (U stands for the dimensional potential) of the electrode relative to the unperturbed plasma, e is the unit charge; $\mu = m_e/m_i$, $\beta = T_e/T_i$ are the ratios of masses and temperatures, respectively, of electrons and molecular ions; $S_i = V_{i,2}/u_{i,2}$ is the ionic velocity ratio, and $u_{i,2}$ is the thermal velocity of molecular ions.

The currents $\bar{I}_p$, $\bar{I}_e$, $\bar{I}_i$ are normalized by the thermal electron current $I_{e,0} = j_{e,0} \cdot S_c$, where $j_{e,0} = en_e u_e/2\sqrt{\pi}$ is the density of the thermal electron current, u_e is the thermal velocity of electrons, S_c and is the area of the collecting surface of the cylindrical electrode.

We assume that the radii of the probe r_p and the reference electrode r_{cp} are subject to the following restrictions [2]:

$$r_p/\lambda_d \le 1, \; r_{cp}/\lambda_d < \xi^* = 3-10,$$

where λ_d is the Debye length in an unperturbed plasma and ξ^* is such value of r_{cp}/λ_d, which limits the applicability of the asymptotic Langmuir solution for the ion current to a cylinder in a transverse flow [4, 5].

Relations (1), (2) that represent the parametric current-voltage characteristic (CVC) of the "probe–plasma–reference electrode" system, include dimensionless parameters η, μ, β, S_i, S_s, φ_{iz}, determined by the parameters of the unperturbed plasma n_e, T_e, m_i, η, T_i, $V_{i,2}$, geometric parameters of the probe system S_p, S_{cp} and the probe bias potential U_{iz}.

The direct problem of probe measurements is to calculate the CVC of the probe $I_p(U_{iz})$ for given parameters of the unperturbed plasma, probe system and bias potential of the probe U_{iz} relative to the reference electrode ($U_{iz} = U_p - U_{cp}$, where U_p, U_{cp} are the potentials of the probe and reference electrode relative to the unperturbed plasma).

For the CVC $\overline{I}_p(\varphi_{iz})$ in the electron saturation region at sufficiently high positive bias potential φ_{iz}, an asymptotic solution is obtained [2]:

$$\overline{I}_p(\varphi_{iz}) \approx \frac{2}{\sqrt{\pi}} \sqrt{\frac{S_s{}^2\mu(1+0.414\eta)^2}{S_s{}^2\mu(1+0.414\eta)^2+1}} \cdot \sqrt{\frac{1/2+S_i^2}{\beta}+\frac{\pi}{4}+\varphi_{iz}} \,. \qquad (3)$$

In the GDS plasma jet, the solution (3) is estimated to be applicable within the potential range:

$$\varphi_{iz}^{\min} < \varphi_{iz} < \varphi_{iz}^{\max},$$

$$\varphi_{iz}^{\min} \approx 6\left[S_s^2\mu(1+0.414\eta)^2+0.14\right]S_i/\sqrt{\beta}+6.5,$$

$$\varphi_{iz}^{\max} << (1+0.414\eta)\left(\mu S_i{}^2/\beta\right)\left(\pi S_{jet}/S_p\right)^2,$$

where S_{jet} is the cross-sectional area of the core region of the jet (region where plasma parameters don't change over the distance from the jet axis).

Within the framework of the accepted mathematical model of current collection (1), (2), in the electron saturation region from the asymptotic solution (3) for dimensional potentials and currents the following relations are derived:

$$I'_p(U_{iz})/I_p(U_{iz}) \approx \frac{1}{\left(1/2+S_i^2\right)kT_i/e+\pi/4\cdot kT_e/e+U_{iz}}, \qquad (4)$$

$$\frac{I_p(U_{iz})\cdot I'_p(U_{iz})}{I_p{}^2(U_{iz,2})-I_p{}^2(U_{iz,1})} \approx \frac{1}{U_{iz,2}-U_{iz,1}}, \; \frac{I_p{}^2(U_{iz,2})+I_p{}^2(U_{iz,1})}{I_p{}^2(U_{iz,2})-I_p{}^2(U_{iz,1})} \approx \frac{U_{iz,2}+U_{iz,1}}{U_{iz,2}-U_{iz,1}}. \quad (5)$$

Here, all bias potentials U_{iz} belong to the range of applicability of solution (3):

$$U_{iz} \in \left[\varphi_{iz}^{\min} kT_e/e, \varphi_{iz}^{\max} kT_e/e \right]. \tag{6}$$

The above relations will be used later in estimating the errors in determining the plasma parameters.

The inverse problem of probe measurements is to determine the values of the plasma parameters by the experimentally obtained CVC $\tilde{I}_p\left(\tilde{U}_{iz}\right)$. The probe system must be able to measure CVC at different electrodes area ratio S_s.

As a result of measurement the probe's current I_p and bias potential U_{iz}, the approximate values are obtained, respectively

$$\tilde{I}_p = I_p\left(1+\tilde{\varepsilon}_I\right),\ \tilde{U}_{iz} = U_{iz}\left(1+\tilde{\varepsilon}_U\right), \tag{7}$$

where $\tilde{\varepsilon}_I$, $\tilde{\varepsilon}_U$ are random values from the ranges $\left[-\varepsilon_I, \varepsilon_I\right]$, $\left[-\varepsilon_U, \varepsilon_U\right]$, respectively; ε_I, ε_U are the limiting relative measurement errors for the corresponding quantities (ε_I, ε_U > 0).

Solution (3) defines the probe current in the electron saturation region as a monotonically increasing function. Then the function $I_p\left(U_{iz}\right)$ is approximated by $\tilde{I}_p\left(\tilde{U}_{iz}\right)$ with a limiting relative error of $\varepsilon_{I,U} = \varepsilon_I + I'_p\left(U_{iz}\right)/I_p\left(U_{iz}\right)\cdot U_{iz}\varepsilon_U$. Taking into account relation (4), we can write

$$\varepsilon_{I,U} = \varepsilon_I + \mathrm{E}\cdot\varepsilon_U, \text{ where } \mathrm{E} = \frac{U_{iz}}{\left(1/2+S_i^2\right)kT_i/e + \pi/4\cdot kT_e/e + U_{iz}} < 1.$$

In the GDS jet, $kT_e/e < S_i^2\, kT_i/e \le U_{iz}$ usually satisfy, so we will not overestimate much if we accept $\mathrm{E}=1$.

Let's consider, based on the asymptotic solution (3), the problems of determining the degree of ion dissociation η and the electron density n_e by the measurements of the probe current in the GDS plasma jet.

The degree of ions dissociation. To determine the degree of dissociation η in the GDS jet, a calculation formula is obtained in [2], which can be written in dimensional form as follows:

$$\eta \approx 2.415\cdot\left(\frac{1}{\sqrt{\mu}\cdot S_{s,2}}\sqrt{\frac{I_{p,1}^2\cdot p_s^2 - I_{p,2}^2}{I_{p,2}^2 - I_{p,1}^2}} - 1\right),\ p_s = S_{s,2}/S_{s,1} > 1. \tag{8}$$

Here, $I_{p,1}$ and $I_{p,2}$ are the probe currents corresponding within the framework of the mathematical model (3) to the bias potential U_{iz} at the values of the ratio of the areas of the probe and the reference electrode of $S_s = S_{s,1}$ and $S_s = S_{s,2}$, respectively. In this case, the bias potential U_{iz} must satisfy condition (6).

Formula (8) determines the degree of dissociation η only in terms of the dimensional currents and does not explicitly depend on the plasma parameters n_e, T_e, S_i, β. In the theory of a single Langmuir probe, there is no such calculation formula [6]. This proves the IPS with cylindrical electrodes to be more informative tool, compared to a single Langmuir probe, of the diagnostics of dissociated laboratory plasma jet.

Let's consider the relative error of the calculation of the degree of dissociation $\overline{\varepsilon}_\eta = (\overline{\eta} - \eta)/\eta$, where $\overline{\eta}$ is values calculated by (8) at exact probe currents $I_{p,1}$, $I_{p,2}$, corresponding to the mathematical model (1), (2), for different values of the area ratio $S_{s,2}$ at p_s = 5. Fig. 1 represents the dependence of $\overline{\varepsilon}_\eta$ on the bias potential U_{iz}. The curves in the figure correspond to $S_{s,2}$ = 100 (1), $S_{s,2}$ = 200 (2), $S_{s,2}$ = 300 (3), $S_{s,2}$ = 400 (4), $S_{s,2}$ = 500 (5), $S_{s,2}$ = 600 (6). The calculations are carried out for $\eta = 0.5$, $S_i = 4$, $\mu = 2 \cdot 10^{-5}$ and $\beta = 4$, which are the parameters of laboratory plasma for simulating the flow conditions in the ionosphere [7]. Within the framework of the current collection model (1) – (2), the error $\overline{\varepsilon}_\eta$ is a methodological error in calculating the degree of dissociation η by (8).

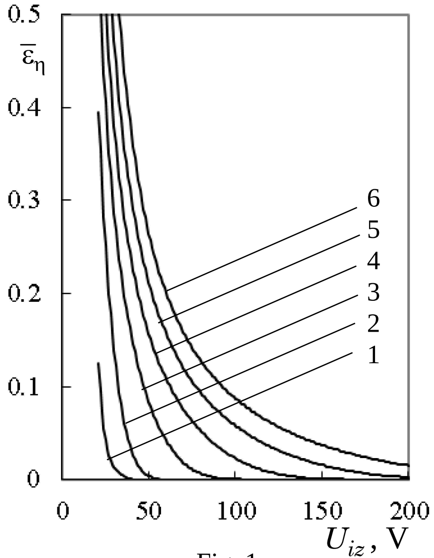

Fig. 1

The obtained results show that an increase in the bias potential U_{iz} and a decrease in the geometric parameter $S_{s,2}$ leads to a monotonous decrease in the methodological error $\overline{\varepsilon}_\eta$. At bias potentials $U_{iz} < 50$ V for all considered values of the parameter $S_{s,2} \geq 100$, the methodological error $\overline{\varepsilon}_\eta$ of formula (8) increases sharply, which makes it difficult to adequately determine the degree of ion dissociation.

At potentials U_{iz} from 50 V to 100 V and $S_{s,2}$ from 100 to 400, the methodological error $\overline{\varepsilon}_\eta$ does not exceed 20 %. At $U_{iz} > 100$ V and $S_{s,2} \geq 100$ the error $\overline{\varepsilon}_\eta$ is significantly smaller and doesn't exceed 10 %.

Let's consider the effect of probe measurement errors and geometric parameters p_s, $S_{s,2}$ on the error in determining the degree of dissociation η by formula (8). Substituting the approximate values (7) into the calculation formula (8) and neglecting the second order small values, after simple transformations, taking into account the solution (3), we can write:

$$\varepsilon_\eta = \sup_{\tilde{\varepsilon}_{I,U}} \left| \frac{\tilde{\eta} - \eta}{\eta} \right| \approx \Psi_1 \cdot \sqrt{\frac{1 + 2\left(\frac{p_s^2 + 1}{p_s^2 - 1} + \frac{2 p_s^2}{p_s^2 - 1} \cdot \frac{1}{S_{s,2}^2 \mu \Psi_2^2} \right) \varepsilon_{I,U}}{1 - 2\left(\frac{p_s^2 + 1}{p_s^2 - 1} + \frac{2}{p_s^2 - 1} \cdot S_{s,2}^2 \mu \Psi_2^2 \right) \varepsilon_{I,U}} - 1}\,, \qquad (9)$$

$$\Psi_1 = 1 + 2.41/\eta\,, \; \Psi_2 = 1 + 0.414\eta\,,$$

where ε_η is the limit value of relative error in determining η, $\tilde{\eta}$ is values calculated by (8) at approximate probe currents $\tilde{I}_{p,1}$, $\tilde{I}_{p,2}$, $\tilde{\varepsilon}_{I,U} = \tilde{\varepsilon}_I + \mathrm{E} \cdot \tilde{\varepsilon}_U$ is the relative error in measuring the probe current.

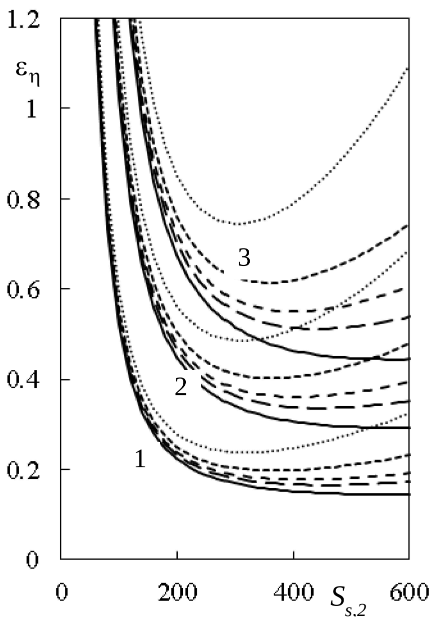

Fig. 2

Fig. 2 and 3 illustrate the influence of the geometric parameters $S_{s,2}$, p_s and limit value of relative error of measuring the probe current $\varepsilon_{I,U}$ on the limit value of relative error ε_η of determining the degree of dissociation η by (8). The results of calculations of ε_η as a dependence on electrode areas ratios $S_{s,2}$ for $\varepsilon_{I,U}$ = 0.01 (curves 1), 0.02 (curves 2), and 0.03 (curves 3). For each value of the measurement error $\varepsilon_{I,U}$, dotted curves correspond to p_s = 3, short dashed curves correspond to p_s = 4, middle dashed curves correspond to p_s = 5, long dashed curves correspond to p_s = 6 and solid curves stand for p_s = 10.

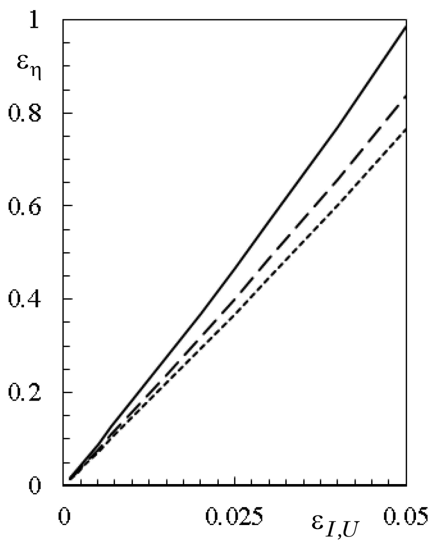

Fig. 3

The calculation results reveal that a decrease in the parameter $S_{s,2}$ from 200 to 100 leads to a sharp increase in the error ε_η for all p_s. As p_s increases, ε_η decreases monotonically for all $\varepsilon_{I,U}$ and $S_{s,2} \geq 100$. For $S_{s,2} \geq 200$ and $p_s \geq 5$, the error ε_η doesn't change significantly.

Taking into account the considered above influence of $S_{s,2}$ and U_{iz} on the methodological error $\bar{\varepsilon}_\eta$, we recommend to take $p_s \geq 5$; $S_{s,2}$ = 200…400 at U_{iz} = 50…100 V and $S_{s,2} \geq 200$ at $U_{iz} > 100$ V for an adequate estimate of the degree of ion dissociation.

Fig. 3 represents the results of calculations of ε_η depending on the measurement error $\varepsilon_{I,U}$ at $S_{s,2}$ = 500 for various p_s = 5 (solid curve), 7 (dashed curve), 10 (dotted curve).

The results obtained allow us to conclude that, in order to adequately determine the degree of ion dissociation, the limit relative error in measuring the probe current and bias potential should not exceed ~ 1 %.

Expanding in relation (9) the square root in a series with respect to $\varepsilon_{I,U}$ and holding 1st order member, we obtain a simpler estimate for the limit value of the relative error of the calculation formula (8)

$$\left|\frac{\tilde{\eta}-\eta}{\eta}\right| \le \varepsilon_\eta \approx \Psi_1\left(1+\frac{1}{S_{s,2}{}^2\mu\cdot\Psi_2{}^2}\right)\left(1+\frac{S_{s,2}{}^2\mu\cdot\Psi_2{}^2+1}{p_s{}^2-1}\right)2\left(\varepsilon_I+\mathrm{E}\varepsilon_U\right).$$

The above estimate, together with results presented in Fig. 1, allows one to select proper geometrical parameters of the IPS and evaluate the required measurement accuracy for an adequate determination of the degree of ion dissociation.

Electron density. By analogy with the case of a single Langmuir probe [6], rewriting relation (3) in dimensional terms, squaring and differentiating with respect to the bias potential U_{iz}, we obtain a calculation formula for the electron density:

$$n_e \approx \frac{\pi}{eS_\mathrm{p}}\sqrt{\frac{m_e}{2e}}\cdot\sqrt{\frac{1}{S_s{}^2\mu\left(1+0.414\eta\right)^2}+1}\cdot\sqrt{\frac{dI_p{}^2(U_{iz})}{dU_{iz}}}. \qquad (10)$$

For a reliable estimate the electron density in the GDS jet using (10), the bias potential U_{iz} must satisfy (6).

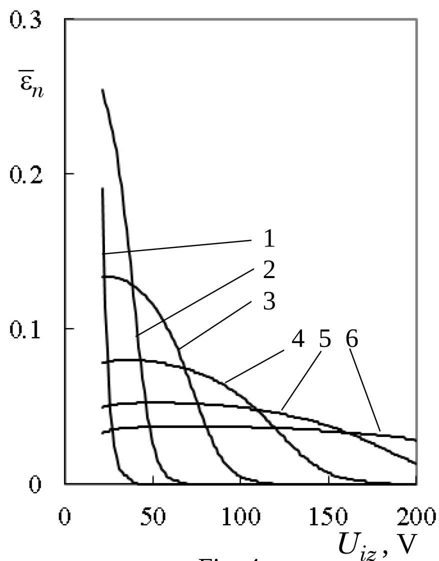

Fig. 4

Fig. 4 shows the dependence on the bias potential U_{iz} of the relative error $\bar{\varepsilon}_n=\left(\bar{n}_e-n_e\right)/n_e$, where $\bar{n}_e$ is the result of calculation by (10) with accurate calculation of the probe current I_p using the mathematical model (1), (2), for various area ratios S_s at p_s = 5. The curves correspond to $S_{s,2}$ = 100 (1), 200 (2), 300 (3), 400 (4), 500 (5) and 600 (6). The calculations were performed for $\eta=0.5$, $S_i=4$, $\mu=2\cdot10^{-5}$ and $\beta=4$. Within the framework of the current collection model (1) – (2) the error $\bar{\varepsilon}_n$ is a methodological error in calculating the electron density n_e using (10).

The results of numerical modeling show that an increase in U_{iz} leads to a monotonic decrease in $\bar{\varepsilon}_n$ for all values of the electrode area ratio $S_s \ge 100$. For all S_s, $\bar{\varepsilon}_n$ reaches its largest value at $U_{iz} < 50$ V. Also, the largest value of $\bar{\varepsilon}_n$ decreases as S_s increases, and at $S_s > 350$ the error $\bar{\varepsilon}_n$ doesn't exceed 10 %. At $S_s = 600$ the methodological error of formula (10) is $\bar{\varepsilon}_n \approx 3$ %, which is quite acceptable for practical use.

Let's consider the influence of errors in probe measurements $\varepsilon_{I,U}$ and a geometrical parameter S_s on the error in determining the electron density n_e using formula (10). Within the framework of model (1)–(2) of probe current collection, similarly to the case of a single cylindrical probe, $I_p^2(U_{iz})$ in the electron saturation region is a linear function. In this case, for any interval $\left[U_{iz,1}, U_{iz,2}\right]$ from the region (6) we can write:

$$\frac{dI_p^2(U_{iz})}{dU_{iz}} = \frac{I_p^2(U_{iz,2}) - I_p^2(U_{iz,1})}{U_{iz,2} - U_{iz,1}},\ U_{iz} \in \left[U_{iz,1}, U_{iz,2}\right].$$

Then the calculation formula (10) writes:

$$n_e \approx \frac{\pi}{eS_\text{p}} \cdot \sqrt{\frac{m_e}{2e}} \cdot \sqrt{\frac{1}{S_s{}^2 \mu (1 + 0.414\eta)^2} + 1} \cdot \sqrt{\frac{I_p^2(U_{iz,2}) - I_p^2(U_{iz,1})}{U_{iz,2} - U_{iz,1}}}. \qquad (11)$$

To estimate the error of determination of the density n_e using formula (11), we substitute approximate values (7) into it. After simple transformations, taking into account the solution (3) and relations (5), neglecting the second order small members, we obtain:

$$\left|\frac{\tilde{n}_e - n_e}{n_e}\right| \le \varepsilon_n \approx \frac{U_{iz,2} + U_{iz,1}}{U_{iz,2} - U_{iz,1}} \cdot \left(\varepsilon_I + \frac{1}{2}\varepsilon_U\right),$$

where ε_n is the limit relative error in determining the density n_e, $\tilde{n}_e$ is values calculated by (11) at approximate probe currents $\tilde{I}_p(\tilde{U}_{iz,1})$, $\tilde{I}_p(\tilde{U}_{iz,2})$, and the bias potentials $U_{iz,1}$, $U_{iz,2}$ belong to the range (6).

As one can see, the relative error ε_n does not explicitly depend on the geometric parameter S_s. However, the total error, including the methodological error of formula (11), which is shown in Fig. 4, does depend on S_s. The resulting estimate of the relative error ε_n, together with results presented in Figs. 4, allows one to select S_s, $U_{iz,1}$, $U_{iz,2}$ and evaluate the required measurement accuracy for an adequate determination of the electron density.

Conclusions. A procedure has been developed for determining the degree of ion dissociation η and the electron density n_e in a supersonic GDS plasma jet by the results of current measurements by IPS with transversely oriented cylindrical electrodes. Within the framework of the accepted assumptions, new calculation formulas are obtained that relate the investigated plasma parameters η and n_e with measured probe currents in the electron saturation region.

The influence of the electrodes current-collecting areas ratio and the probe bias potential relative to the reference electrode potential on the determination of plasma parameters are studied numerically using the current-collection model in the electron saturation region. Within the framework of the mentioned model, the errors in the determination of plasma parameters are estimated analytically depending on the geometric parameters of the probe system, the measurement accuracy of the probe currents, and the probe bias potentials relative to the potential of the reference electrode.

The obtained numerical and analytical estimates of the degree of ion dissociation and electron density determination errors help to select the geometric parameters of the probe system and the required measurement accuracy when planning and carrying out experiments on laboratory plasma diagnostics.

1. *Gabovich M. D.* Physics and Technology of Plasma Ion Sources. Moscow: Atomizdat, 1972. 304 pp. (*in Russian*).
2. *Lazuchenkov D. N., Lazuchenkov N. M.* Determination of parameters of a dissociated supersonic rarefied plasma flow by current-voltage characteristics of isolated system of cylindrical probes. Teh. Meh. 2020. No. 2. Pp. 80–88. https://doi.org/10.15407/itm2020.02.080
3. *Sapozhnikov G. I.* Experimental studies of the flow of accelerated ions and its interaction with models. Uchenye Zapiski TsAGI. 1971. V. 2. No. 1. Pp. 129–133. (*in Russian*).
4. *Godard R., Laframboise J.* Total current to cylindrical collectors in collision less plasma flow. Space Science. 1983. V. 31. No. 3. Pp. 275–283. https://doi.org/10.1016/0032-0633(83)90077-6
5. *Lazuchenkov D. N., Lazuchenkov N. M.* Calculation of the ion current to a conducting cylinder in a supersonic flow of a collisionless plasma. Teh. Meh. 2022. No. 3. Pp. 91–98. https://doi.org/10.15407/itm2022.03.091
6. *Chung P. M., Talbot L., Touryan K. J.* Electric Probes in Stationary and Flowing Plasmas. Springer-Verlag, 1975. 150 pp. https://doi.org/10.1007/978-3-642-65886-0
7. *Shuvalov V. A., Pis'mennyi N. I., Lazuchenkov D. N., Kochubey G. S.* Probe diagnostics of laboratory and ionospheric rarefied plasma flows. Instruments and Experimental Techniques. 2013. V. 56. No. 4. Pp. 459–467. https://doi.org/10.1134/S002044121304009X

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