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MODEL OF E-POLARIZED WAVE PROPAGATION IN THE MULTILAYER DIELECTRIC STRUCTURE

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В роботі розглядається питання визначення діелектричної проникності багатошарових діелектричних структур при використанні радіохвильових інтерференційних методів. Відзначено, що у загальному випадку при проведенні вимірів інтерференційними методами одному виміряному значенню коефіцієнта відбиття може відповідати безліч значень діелектричної проникності. Ця невизначеність може бути усунена, якщо є можливість попереднього визначення впливу на коефіцієнти відбиття різних параметрів зондуючих електромагнітних хвиль. Зокрема є важливим отримання попередньої оцінки пливу кутів падіння та поляризації на діапазон зміни коефіцієнтів відбиття при зміні одного з параметрів структур.

В даній роботі розглядається випадок, коли на багатошарову структуру падає плоска електромагнітна хвиля, у якій магнітне поле перпендикулярно площині падіння, тобто з Е-поляризацією. Мета роботи – розробка моделі поширення Е-поляризованої електромагнітної хвилі скрізь багатошарову діелектричну структуру при довільному куті падіння і визначення межі діапазону зміни коефіцієнта відбиття при зміні діелектричної проникності її шарів. Розроблено модель розповсюдження Е-поляризованої електромагнітної хвилі в двошаровій діелектричній структурі, яку розташовано на металевій основі з ідеальною провідністю при довільному куті падіння електромагнітної хвилі на неї з повітря. На основі розробленої моделі запропоновано метод вимірювання відносної діелектричної проникності та тангенсу кута діелектричних втрат. Показано, що при нормальному падінні хвилі модуль коефіцієнта відбиття для Н- и Е-поляризації однакові. Тому для визначення відносної діелектричної проникності та тангенсу кута діелектричних втрат по виміряному модулю коефіцієнта відбиття необхідно проводити не тільки при нормальному, но й скісному падінні електромагнітної хвилі, при якому модулі коефіцієнта відбиття будуть відрізнятися для випадків Н- и Е-поляризації.

Ключові слова: *багатошарові діелектричні структури, Е-поляризація, Н-поляризація, діелектрична проникність, коефіцієнт відбиття.*

This paper addresses the determination of the dielectric constant of multilayer dielectric structures by radiowave interferometry. In the general case, in interferometry measurements to one measured value of the reflection coefficient there may correspond an infinity of dielectric constants. This ambiguity may be resolved by first determining the effect of different parameters of the probing electromagnetic wave on the reflection coefficient. In particular, it is important to have a preliminary estimate of the effect of the incidence angle and the polarization on the range of variation of the reflection coefficient with the variation of one of the structure parameters.

This paper considers the case where a plane E-polarized electromagnetic wave, i.e. a wave whose magnetic field is perpendicular to the incidence plane, is incident on a multilayer dielectric structure. The aim of this work is to develop a model of the propagation of an E-polarized electromagnetic wave through a multilayer dielectric structure at an arbitrary incidence angle and to determine the range of variation of the reflection coefficient with the variation of the dielectric constants of the layers. The paper presents a model of the propagation of an E-polarized electromagnetic wave in a two-layer dielectric structure. A metal base, which is an ideal conductor, underlies the structure. The electromagnetic wave is incident from the air at an arbitrary incidence angle. Based on the model, a method is proposed for measuring the relative dielectric constant and the dielectric loss tangent. It is shown that at a normal incidence the reflection coefficient magnitude is the same both for H- and E-polarization. Because of this, determining the relative dielectric constant and the loss tangent from the measured reflection coefficient magnitude calls for measurements not only at a normal incidence, but also at an oblique incidence, at which the reflection coefficient magnitudes will be different for H- and E-polarization.

Keywords: *multilayer dielectric structures, E-polarization, H-polarization, dielectric constant, reflection coefficient.*

The current level of development of interference microwave [1-8] material control techniques shows their great potential for measuring the properties of multi-layer dielectric constant.

Work [8, 9] noted that, in general, many dielectric constant values can correspond to one measured coefficient of reflection when measuring multilayer constant.

Therefore, it is very important to obtain a preliminary estimate of the impact of the angle of incidence and polarization on the range of changes in reflection coefficient when one of the structure parameters changes.

In [9], we considered the case of the case of a drop of a flat electromagnetic wave into a multilayer structure in which the magnetic field is perpendicular to the plane of the drop with E-polarized.

To obtain a complete estimate of the influence of polarization on the reflection coefficient, in this work, we consider the case when a plane electromagnetic wave is incident on a multilayer structure, in which the magnetic field is perpendicular to the plane of incidence with electron polarization.

The aim of this work is to develop a model of the propagation of an E-polarized electromagnetic wave through a multilayer dielectric structure at an arbitrary incidence angle and to determine the range of variation of the reflection coefficient with the variation of the dielectric constants of the layers.

In order to develop an e-polarized electromagnetic wave propagation model, the same layered structure was considered as in the work [9].

It is presented in figure 1. This structure consists of two layers of dielectric located on the surface of the metal. Layer 1 has thickness d_1 and dielectric constant e_1. Layer 2 has thickness d_2 and dielectric permeability e_2.

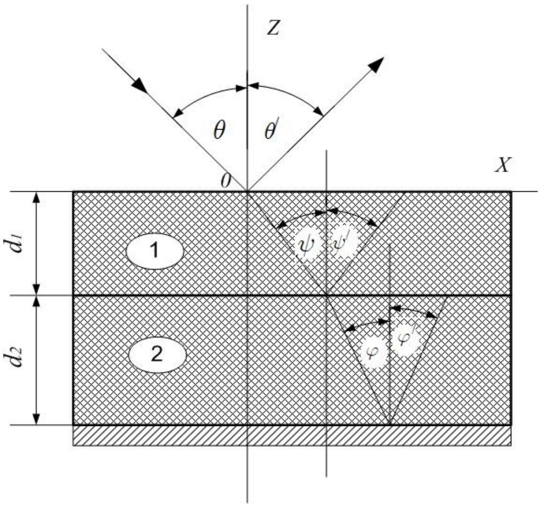

Fig. 1

Let the angle of incidence of the electromagnetic wave (the angle between the normal and the wavefront of the incident wave and perpendicular to the plane of the partition of the media) be equal ¯ . Enter the same coordinate system as in [9]. However, now the OY axis will be directed along the vector of the electric field of the incident wave.

In the case in question, the vector of the electric field of a wave at any point at the coordinates *x*, *y*, *z* has a single component E_y, which is expressed through the constant propagation of the electromagnetic wave in the medium γ and the distance from the origin to the wave surface ξ

$$E_y(x,y,z) = E_0 e^{-¥¶(x,y,z)}$$

where E_0 – amplitude factor; ¥ – propagation constant

The distance from the origin to the wave surface equals the projection of the radius-vector $\vec{r}$, which is drawn from the origin to a point on the wave surface at the coordinates *x*, *y*, and *z* on the direction of the normal to the wave surface.

Therefore, in this case, the value ξ is defined by the expression

$$\xi = x \sin \theta + z \cos \theta .$$

This results in an equation for the incident wave

$$E_{y0}^{+} = E_0^{+} e^{-¥_0 (x \cdot \sin ¯ + z \cdot \cos ¯)}$$

where E_{y0}^{+} – Y-component of the electric field of the incident wave in the air; E_0^{+} – amplitude of the electric field of the incident wave in the air.

A similar approach to air-reflected wave gives the following equation

$$E_{y0}^{-} = E_0^{-} e^{-¥_0 (x \cdot \sin ¯' - z \cdot \cos ¯')},$$

where E_{y0}^{-} – Y-component of electric field vector of the reflected wave in air; E_0^{-} – amplitude of the electric field of the reflected wave in air.

The resulting electric field E_0 in the air is the superposition of the incident and reflected waves

$$E_0 = E_0^{+} e^{-¥_0 (x \cdot \sin ¯ + z \cdot \cos ¯)} + E_0^{-} e^{-¥_0 (x \cdot \sin ¯' - z \cdot \cos ¯')} .$$

By analogy, it is possible to write the expressions for the resulting electric field in the first and second dielectrics:

$$E_1 = E_1^{+} e^{-¥_1 (x \sin \cdot + z \cdot \cos \cdot)} + E_1^{-} e^{-¥_1 (x \sin \cdot - z \cos \cdot)}, \quad (1)$$

$$E_2 = E_2^{+} e^{-¥_2 (x \cdot \sin ¨ + z \cdot \cos ¨)} + E_2^{-} e^{-¥_2 (x \cdot \sin ¨ - z \cdot \cos ¨)} \quad (2)$$

where E_1, E_1^{+}, E_1^{-} – the resultant electric field, the electric field amplitude of the incident wave and the electric field amplitude of the reflected wave in the first layer; E_2, E_2^{+}, E_2^{-} – the resultant electric field, the electric field amplitude of the incident wave and the electric field amplitude of the reflected wave in the second dielectric.

The relationship between angles · and ¨ in equations (1) and (2) in the first and second dielectrics and the angle ¯ was given by the ratios obtained in [9]:

$$\sin y = \frac{g_0}{g_1} \sin q , \quad \sin j = \frac{g_0}{g_2} \sin q .$$

We use boundary conditions for tangential components of magnetic and electric fields on the boundary surfaces of the studied media for to derive algebraic equations for unknown amplitudes E_n^{+} and E_n^{-} ($n = 0, 1, 2$).

The condition of the tangential components of the electric field in the plane $z = 0$ is written as

$$E_0^{+} e^{-¥_0 x \sin ¯} + E_0^{-} e^{-¥_0 x \sin ¯} = E_1^{+} e^{-¥_1 x \sin \cdot} + E_1^{-} e^{-¥_1 x \sin \cdot} . \quad (3)$$

Considering the equation $¥_0 \sin ¯ = ¥_1 \sin ·$, obtained in [9], the equation (3) can be written as

$$E_0^+ + E_0^- = E_1^+ + E_1^- . \quad (4)$$

The expressions for the components of the magnetic field can be obtained from Maxwell’s equation for a monochromatic electromagnetic wave with a circular frequency $µ$:

$$\text{rot} E = - j « µ H \quad (5)$$

where E – the complex vector amplitude of electric field; H – the complex vector amplitude of magnetic field.

When considering the continuity conditions at the boundary of the layers, it is sufficient to consider only the tangential component of the magnetic field H_x .

It follows from equation (5) that an expression for H_x can be written in terms of E_y :

$$H_x = \frac{1}{jµ«} \frac{\partial E_y}{\partial z} . \quad (6)$$

Here and further we shall assume that the magnetic constant of the air, first and second layer equals the magnetic permeability of the vacuum $µ_0$.

Considering equation (6), we have:

$$H_0 = -\frac{¥_0 \cos ¯}{jµ«_0} \left[E_0^+ e^{-¥_0 (x \sin ¯ + z \cos ¯)} - E_0^- e^{-¥_0 (x \sin ¯ - z \cos ¯)} \right], \quad (2.12)$$

$$H_1 = -\frac{¥_1 \cos ·}{jµ«_0} \left[E_1^+ e^{-¥_1 (x \sin · + z \cos ·)} - E_1^- e^{-¥_1 (x \sin · - z \cos ·)} \right], \quad (2.13)$$

$$H_2 = -\frac{¥_2 \cos ¨}{jµ«_0} \left[E_2^+ e^{-¥_2 (x \sin ¨ + z \cos ¨)} - E_2^- e^{-¥_2 (x \sin ¨ - z \cos ¨)} \right] \quad (2.14)$$

where H_0, H_1, H_2 – the resulting magnetic field in the air, the first and the second media.

The third medium is a perfectly conductive metal, so the electric field E_3 in it is zero.

The requirement of continuity of the tangential component of the electric field in the plane $z = d_1 + d_2$, which has the form $E_2 = E_3$, is reduced to

$$E_2^+ e^{-¥_2 (d_1 + d_2) \cos ¨} + E_2^- e^{¥_2 (d_1 + d_2) \cos ¨} = 0 . \quad (7)$$

From equation (7) for the coefficient of reflection R_2 at the border of the second dielectric and metal layer we obtain

$$R_2 = \frac{H_2^-}{H_2^+} = -e^{-2¥_2 (d_1 + d_2) \cos ¨} . \quad (8)$$

For a plane $z = d_1$ of tangential components $E_1 = E_2$ and $H_1 = H_2$, the following pair of equations are obtained:

$$E_1^+ e^{-\gamma_1 d_1 \cos\psi} + E_1^- e^{\gamma_1 d_1 \cos\psi} = E_2^+ e^{-\gamma_2 d_1 \cos\varphi} + E_2^- e^{\gamma_2 d_1 \cos\varphi}, \tag{9}$$

$$\gamma_1 \cos\psi \left(E_1^+ e^{-\gamma_1 d_1 \cos\psi} - E_1^- e^{\gamma_1 d_1 \cos\psi} \right) = \gamma_2 \cos\varphi \left(E_2^+ e^{-\gamma_2 d_1 \cos\varphi} - E_2^- e^{\gamma_2 d_1 \cos\varphi} \right). \tag{10}$$

Dividing (10) by (9) and considering the (8) the expression for the coefficient of reflection R_1 in the plane $z = d_1$, i.e. at the border of the first and second dielectric

$$R_1 = \frac{E_1^-}{E_1^+} = \frac{1-q_1}{1+q_1} e^{-2\gamma_1 d_1 \cos\psi} \tag{11}$$

where

$$q_1 = \frac{\gamma_2 \cos\varphi}{\gamma_1 \cos\psi} \left(\frac{1 - R_2 e^{2\gamma_2 d_1 \cos\varphi}}{1 + R_2 e^{2\gamma_2 d_1 \cos\varphi}} \right). \tag{12}$$

On the boundary of the air and the first dielectric, boundary conditions for the magnetic field have the form $H_0 = H_1$, from which follows

$$\gamma_0 \cos\theta \, (E_0^+ - E_0^-) = \gamma_1 \cos\psi \, (E_1^+ - E_1^-). \tag{13}$$

Dividing (13) by (4) and considering the (11) the expression for the coefficient of reflection $R_0 = E_0^- / E_0^+$

$$R_0 = \frac{1-q_0}{1+q_0} \tag{14}$$

where

$$q_0 = \frac{\gamma_1 \cos\psi}{\gamma_0 \cos\theta} \left(\frac{1-R_1}{1+R_1} \right). \tag{15}$$

The functions $\cos\psi$ and $\cos\varphi$, which are included in the ratios for reflection coefficients R_0, R_1, R_2 , can be represented by a given angle of incidence θ, as shown in paper [9]:

$$\cos\psi = \sqrt{1 - \left(\frac{\gamma_0}{\gamma_1}\right)^2 \sin^2\theta}, \quad \cos\varphi = \sqrt{1 - \left(\frac{\gamma_0}{\gamma_2}\right)^2 \sin^2\theta}.$$

The joint solution of the expressions (8), (11), (12), (14), (15) makes it possible to find a reflective coefficient of the structure under consideration on the basis of the parameters of the multilayer structure and the angle of the incident of the electromagnetic wave on it.

In order to evaluate the possibilities of the model, calculations were made to determine the relationship of the reflection rate module to the angle of incident θ for the relative dielectric constant ε_2' of the variable range from 2 to 4.

On fig.2 the calculated dependencies of the coefficient of reflection module on the angle of incident ¯ for the three $£_2' = 2$, $£_2' = 3$, $£_2' = 4$ relative dielectric constant values are shown.

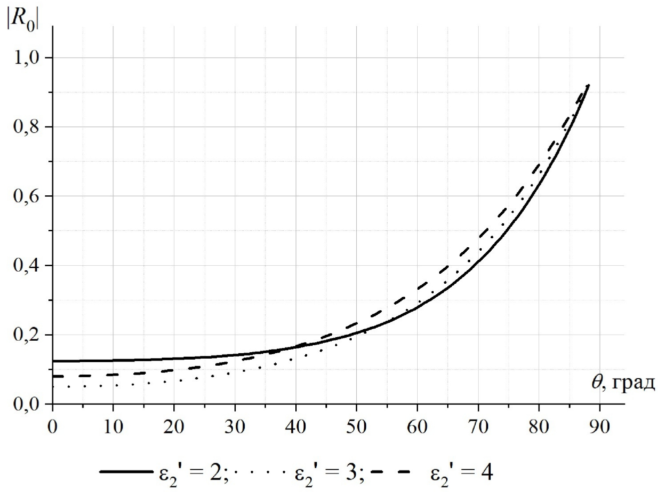

Fig. 2

As can be seen from Figure 2, with E-polarized, the widest range of modulus change in the coefficient of reflection at a change in the relative dielectric constant of the second dielectric corresponds to the normal incidence of the electromagnetic wave (¯ = 0).

Comparison of these results with the results for the H-polarization coefficient reflection R_0^H in research [9] (fig. 3), shows that the intersection points of the curves corresponding to the different relative dielectric constant values $£_2'$, for H and E- polarized correspond to different incidence angles ¯.

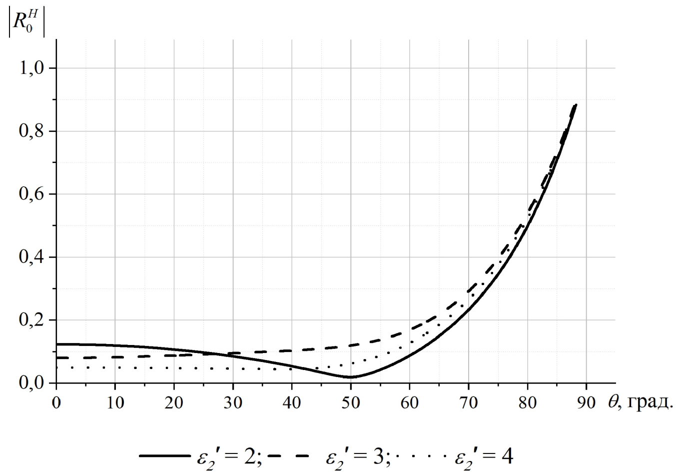

Fig. 3

The comparison of the results presented in Fig. 2 and Fig. 3 also shows that for θ = 0, the modules of the reflection coefficients for H and E- for H and E-polarized are the same.

We will prove that this fact is true in the general case.

Indeed, in (8) and in equation $R_2 = \frac{H_2^-}{H_2^+} = e^{-2g_2(d_1+d_2)\cos j}$ from [9] follows that $R_{2E} = -R_{2H}$ (hereinafter the lower index E denotes the corresponding value for E-polarized, and the lower index H denotes the H-polarized).

If ¯ = 0 the functions $\cos ¨ = \cos · = 1$ therefore the expressions obtained in [9] $q_1 = \frac{£_1¥_2 \cos ¨}{£_2¥_1 \cos ·}\left(\frac{1-R_2 e^{2¥_2 d_1 \cos ¨}}{1+R_2 e^{2¥_2 d_1 \cos ¨}}\right)$, and equation (12) take the form:

$$q_{1H} = \frac{£_1¥_2}{£_2¥_1}\left(\frac{1-R_{2H} e^{2¥_2 d_1}}{1+R_{3H} e^{2¥_2 d_1}}\right), \tag{16}$$

$$q_{1E} = \frac{¥_2}{¥_1}\left(\frac{1-R_{2E} e^{2¥_2 d_1}}{1+R_{2E} e^{2¥_2 d_1}}\right). \tag{17}$$

Considering the definition for propagation constant [8], for the ratio $\frac{£_1¥_2}{£_2¥_1}$ in (16) we obtain

$$\frac{£_1¥_2}{£_2¥_1} = \frac{¥_1}{¥_2}. \tag{18}$$

Considering equation (18) and equality $R_{2E} = -R_{2H}$ for q_{1H}:

$$q_{1H} = \frac{¥_1}{¥_2}\left(\frac{1+R_{2E} e^{2¥_2 d_1}}{1-R_{2E} e^{2¥_2 d_1}}\right) = \frac{1}{q_{1E}}. \tag{19}$$

Put up the (19) to the equation $R_1 = \frac{H_1^-}{H_1^+} = \frac{1-q_1}{1+q_1} e^{-2g_1 d_1 \cos y}$ obtained in [9].

Compare the result with (11). It follows that there is equality $R_{1E} = -R_{1H}$.

Similarly, it can be shown that $q_{1H} = \frac{1}{q_{1E}}$ and $R_{1E} = -R_{1H}$ i.e. $|R_{1E}| = |R_{1H}|$.

Thus, it is shown that, in a normal incidence of a wave, the module of the coefficient of reflection for H and E-polarized is the same.

Therefore, in order to determine the relative dielectric constant and dissipation factor, the coefficient of reflection of the measurement should be carried out not only at the normal incidence but also at the inclination incidence of the electromagnetic wave, when the reflectance modules will be different for H and E-polarized.

Conclusions. The paper presents the model of the propagation of an E-polarized electromagnetic wave in the two-layer dielectric structure. The model allows one to estimate the reflection coefficient of the structure as a function of its parameters and the incidence angle in case the electromagnetic wave is incident from the air at an arbitrary incidence angle.

It is shown that, in order to determine the relative dielectric constant and dissipation factor, the coefficient of reflection should be defined not only at the normal incidence but also at the inclination incidence of the electromagnetic wave, when the reflectance modules will be different for H and E-polarization.

Based on this model as well as the model described in work [9], a method for measuring relative dielectric constant and dissipation factor was proposed.

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